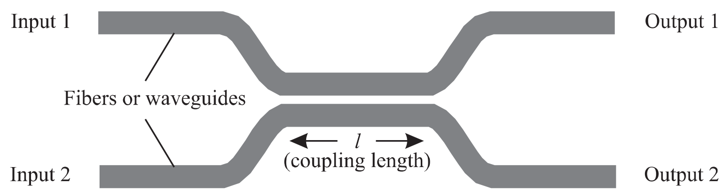

Figure 3.1 A directional coupler. The coupler is typically built by fusing two fibers together. It can also be built using waveguides in integrated optics.

3.1 Couplers

A *directional coupler* is used to combine and split signals in an optical network. A 2×2 coupler consists of two input ports and two output ports, as is shown in Figure 3.1. The most commonly used couplers are made by fusing two fibers together in the middle—these are called fused fiber couplers. Couplers can also be fabricated using waveguides in integrated optics. A 2×2 coupler, shown in Figure 3.1, takes a fraction α of the power from input 1 and places it on output 1 and the remaining fraction $1 - \alpha$ on output 2. Similarly, a fraction $1 - \alpha$ of the power from input 2 is distributed to output 1 and the remaining power to output 2. We call α the coupling ratio.

The coupler can be designed to be either wavelength selective or wavelength independent (sometimes called wavelength flat) over a usefully wide range. In a wavelength-independent device, α is independent of the wavelength; in a wavelength-selective device, α depends on the wavelength.

A coupler is a versatile device and has many applications in an optical network. The simplest application is to combine or split signals in the network. For example, a coupler can be used to distribute an input signal equally among two output ports if the coupling length, l in Figure 3.1, is adjusted such that half the power from each input appears at each output. Such a coupler is called a 3 dB coupler. An $n \times n$ star coupler is a natural generalization of the 3 dB 2×2 coupler. It is an n-input, n-output device with the property that the power from each input is divided equally among all the outputs. An $n \times n$ star coupler can be constructed by suitably interconnecting a number of 3 dB couplers, as shown in Figure 3.2. A star coupler is useful when multiple signals need to be combined and broadcast to many outputs. However, other constructions of an $n \times n$ coupler in integrated optics are also possible (see, for example, [Dra89]).

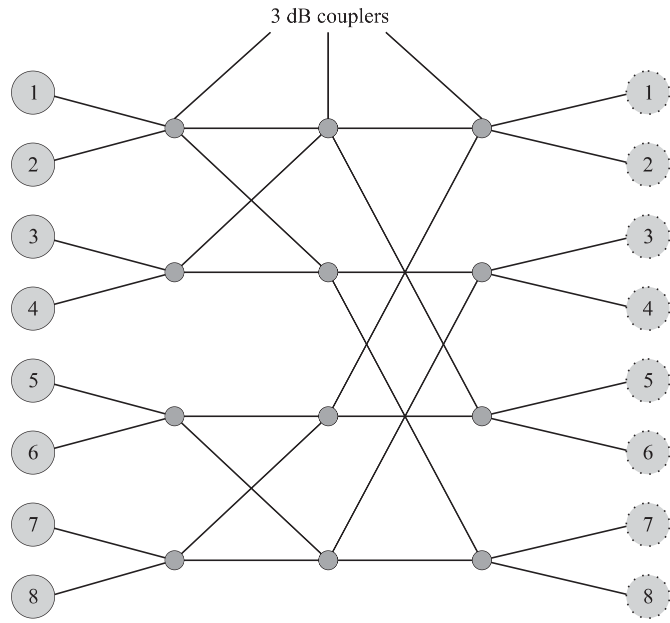

Figure 3.2 A star coupler with eight inputs and eight outputs made by combining 3 dB couplers. The power from each input is split equally among all the outputs.

Couplers are also used to tap off a small portion of the power from a light stream for monitoring purposes or other reasons. Such couplers are also called taps and are designed with values of α close to 1, typically 0.90–0.95.

Couplers are the building blocks for several other optical devices. We will explore the use of directional couplers in modulators and switches in Sections 3.5.4 and 3.7. Couplers are also the principal components used to construct *Mach-Zehnder interferometers,* which can be used as optical filters, multiplexers/demultiplexers, or as building blocks for optical modulators, switches, and wavelength converters. We will study these devices in Section 3.3.7.

So far, we have looked at wavelength-independent couplers. A coupler can be made wavelength selective, meaning that its coupling coefficient will then depend on the wavelength of the signal. Such couplers are widely used to combine signals at 1310 nm and 1550 nm into a single fiber without loss. In this case, the 1310 nm signal on input 1 is passed through to output 1, whereas the 1550 nm signal on input 2 is passed through also to output 1. The same coupler can also be used to separate the two signals coming in on a common fiber. Wavelength-dependent couplers are also used to combine 980 nm or 1480 nm pump signals along with a 1550 nm signal into an erbium-doped fiber amplifier; see Figures 3.34 and 3.37.

In addition to the coupling ratio α, we need to look at a few other parameters while selecting couplers for network applications. The *excess loss* is the loss of the device above the fundamental loss introduced by the coupling ratio α. For example, a 3 dB coupler has a nominal loss of 3 dB but may introduce additional losses of, say, 0.2 dB. The other parameter is the variation of the coupling ratio α compared to its nominal value, due to tolerances in manufacturing, as well as wavelength dependence. In addition, we also need to maintain low *polarization-dependent loss* (PDL) for most applications.

3.1.1 Principle of Operation

When two waveguides are placed in proximity to each other, as shown in Figure 3.1, light "couples" from one waveguide to the other. This is because the propagation modes of the combined waveguide are quite different from the propagation modes of a single waveguide due to the presence of the other waveguide. When the two waveguides are identical, which is the only case we consider in this book, light launched into one waveguide couples to the other waveguide completely and then back to the first waveguide in a periodic manner. A quantitative analysis of this coupling phenomenon must be made using *coupled mode theory* [Yar97] and is beyond the scope of this book. The net result of this analysis is that the electric fields, E_{o1} and E_{o2}, at the outputs of a directional coupler may be expressed in terms of the electric fields at the inputs E_{i1} and E_{i2}, as follows:

$$\begin{pmatrix} E_{o1}(f) \\ E_{o2}(f) \end{pmatrix} = e^{-i\beta l} \begin{pmatrix} \cos(\kappa l) & i\sin(\kappa l) \\ i\sin(\kappa l) & \cos(\kappa l) \end{pmatrix} \begin{pmatrix} E_{i1}(f) \\ E_{i2}(f) \end{pmatrix}. \tag{3.1}$$

Here, l denotes the coupling length (see Figure 3.1), and β is the propagation constant in each of the two waveguides of the directional coupler. The quantity κ is called the *coupling coefficient* and is a function of the width of the waveguides, the refractive indices of the waveguiding region (core) and the substrate, and the proximity of the two waveguides. Equation (3.1) will prove useful in deriving the transfer functions of more complex devices built using directional couplers (see Problem 3.1).

Although the directional coupler is a two-input, two-output device, it is often used with only one active input, say, input 1. In this case, the power transfer function of the directional coupler is

$$\begin{pmatrix} T_{11}(f) \\ T_{12}(f) \end{pmatrix} = \begin{pmatrix} \cos^2(\kappa l) \\ \sin^2(\kappa l) \end{pmatrix}. \tag{3.2}$$

Here, $T_{ij}(f)$ represents the power transfer function from input i to output j and is defined by $T_{ij}(f) = |E_{oj}|^2/|E_{ii}|^2$. Equation (3.2) can be derived from (3.1) by setting $E_{i2} = 0$.

Note from (3.2) that for a 3 dB coupler the coupling length must be chosen to satisfy $\kappa l = (2k+1)\pi/4$, where k is a nonnegative integer.

3.1.2 Conservation of Energy

The general form of (3.1) can be derived merely by assuming that the directional coupler is lossless. Assume that the input and output electric fields are related by a general equation of the form

$$\begin{pmatrix} E_{o1} \\ E_{o2} \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix}. \tag{3.3}$$

The matrix

$$\mathbf{S} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}$$

is the transfer function of the device relating the input and output electric fields and is called the *scattering matrix*. We use complex representations for the input and output electric fields, and thus the s_{ij} are also complex. It is understood that we must consider the real part of these complex fields in applications. This complex representation for the s_{ij} allows us to conveniently represent any induced phase shifts.

For convenience, we denote $\mathbf{E}_o = (E_{o1}, E_{o2})^T$ and $\mathbf{E}_i = (E_{i1}, E_{i2})^T$, where the superscript T denotes the transpose of the vector/matrix. In this notation, (3.3) can be written compactly as $\mathbf{E}_o = \mathbf{S}\mathbf{E}_i$.

The sum of the powers of the input fields is proportional to $\mathbf{E}_i^T\mathbf{E}_i^* = |E_{i1}|^2 + |E_{i2}|^2$. Here, * represents the complex conjugate. Similarly, the sum of the powers of the output fields is proportional to $\mathbf{E}_o^T\mathbf{E}_o^* = |E_{o1}|^2 + |E_{o2}|^2$. If the directional coupler is lossless, the power in the output fields must equal the power in the input fields so that

$$\begin{aligned} \mathbf{E}_o^T\mathbf{E}_o &= (\mathbf{S}\mathbf{E}_i)^T(\mathbf{S}\mathbf{E}_i)^* \\ &= \mathbf{E}_i^T(\mathbf{S}^T\mathbf{S}^*)\mathbf{E}_i^* \\ &= \mathbf{E}_i^T\mathbf{E}_i^*. \end{aligned}$$

Since this relationship must hold for arbitrary $\mathbf{E}_i$, we must have

$$\mathbf{S}^T\mathbf{S}^* = \mathbf{I}, \tag{3.4}$$

where **I** is the identity matrix. Note that this relation follows merely from conservation of energy and can be readily generalized to a device with an arbitrary number of inputs and outputs.

For a 2 × 2 directional coupler, by the symmetry of the device, we can set $s_{21} = s_{12} = a$ and $s_{22} = s_{11} = b$. Applying (3.4) to this simplified scattering matrix, we get

$$|a|^2 + |b|^2 = 1 \tag{3.5}$$

and

$$ab^* + ba^* = 0. \tag{3.6}$$

From (3.5), we can write

$$|a| = \cos(x) \text{ and } |b| = \sin(x). \tag{3.7}$$

If we write $a = \cos(x)e^{i\phi_a}$ and $b = \sin(x)e^{i\phi_b}$, (3.6) yields

$$\cos(\phi_a - \phi_b) = 0. \tag{3.8}$$

Thus ϕ_a and ϕ_b must differ by an odd multiple of $\pi/2$. The general form of (3.1) now follows from (3.7) and (3.8).

The conservation of energy has some important consequences for the kinds of optical components that we can build. First, note that for a 3 dB coupler, though the electric fields at the two outputs have the same magnitude, they have a relative phase shift of $\pi/2$. This relative phase shift, which follows from the conservation of energy as we just saw, plays a crucial role in the design of devices such as the Mach-Zehnder interferometer that we will study in Section 3.3.7.

Another consequence of the conservation of energy is that *lossless combining* is not possible. Thus we cannot design a device with three ports where the power input at two of the ports is completely delivered to the third port. This result is demonstrated in Problem 3.2.

3.2 Isolators and Circulators

Couplers and most other passive optical devices are *reciprocal* devices in that the devices work exactly the same way if their inputs and outputs are reversed. However, in many systems there is a need for a passive *nonreciprocal* device. An *isolator* is an example of such a device. Its main function is to allow transmission in one direction through it but block all transmission in the other direction. Isolators are used in systems at the output of optical amplifiers and lasers primarily to prevent reflections from entering these devices, which would otherwise degrade their performance. The

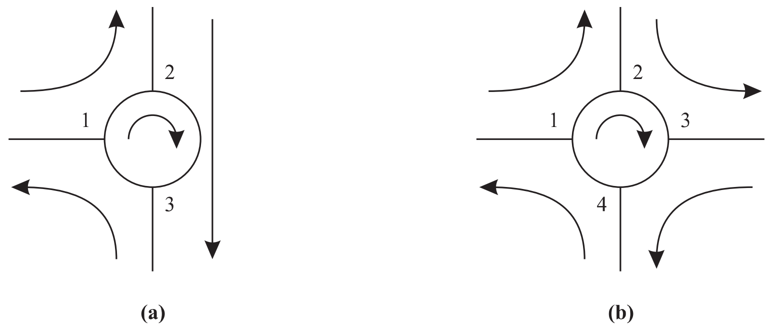

Figure 3.3 Functional representation of circulators: (a) three-port and (b) four-port. The arrows represent the direction of signal flow.

two key parameters of an isolator are its *insertion loss*, which is the loss in the forward direction and which should be as small as possible, and its *isolation*, which is the loss in the reverse direction and which should be as large as possible. The typical insertion loss is around 1 dB, and the isolation is around 40–50 dB.

A *circulator* is similar to an isolator, except that it has multiple ports, typically three or four, as shown in Figure 3.3. In a three-port circulator, an input signal on port 1 is sent out on port 2, an input signal on port 2 is sent out on port 3, and an input signal on port 3 is sent out on port 1. Circulators are useful to construct optical add/drop elements, as we will see in Section 3.3.4. Circulators operate on the same principles as isolators; therefore we only describe the details of how isolators work next.

3.2.1 Principle of Operation

In order to understand the operation of an isolator, we need to understand the notion of *polarization*. Recall from Section 2.3.3 that the *state of polarization* (SOP) of light propagating in a single-mode fiber refers to the orientation of its electric field vector on a plane that is orthogonal to its direction of propagation. At any time, the electric field vector can be expressed as a linear combination of the two orthogonal linear polarizations supported by the fiber. We will call these two polarization modes the horizontal and vertical modes.

The principle of operation of an isolator is shown in Figure 3.4. Assume that the input light signal has the vertical SOP shown in the figure. It is passed through a *polarizer*, which passes only light energy in the vertical SOP and blocks light energy in the horizontal SOP. Such polarizers can be realized using crystals, called *dichroics,*

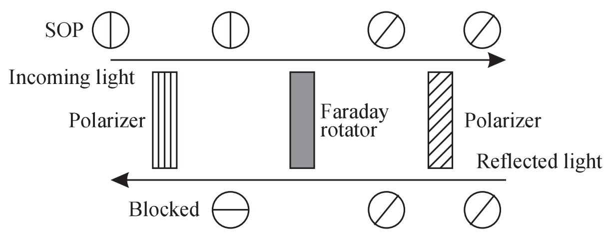

Figure 3.4 Principle of operation of an isolator that works only for a particular state of polarization of the input signal.

which have the property of selectively absorbing light with one SOP. The polarizer is followed by a *Faraday rotator*. A Faraday rotator is a nonreciprocal device, made of a crystal that rotates the SOP, say, clockwise, by 45°, regardless of the direction of propagation. The Faraday rotator is followed by another polarizer that passes only SOPs with this 45° orientation. Thus the light signal from left to right is passed through the device without any loss. On the other hand, light entering the device from the right due to a reflection, with the same 45° SOP orientation, is rotated another 45° by the Faraday rotator, and thus blocked by the first polarizer.

Note that the preceding explanation assumes a particular SOP for the input light signal. In practice we cannot control the SOP of the input, and so the isolator must work regardless of the input SOP. This requires a more complicated design, and many different designs exist. One such design for a miniature polarization-independent isolator is shown in Figure 3.5. The input signal with an arbitrary SOP is first sent through a *spatial walk-off polarizer* (SWP). The SWP splits the signal into its two orthogonally polarized components. Such an SWP can be realized using *birefringent* crystals whose refractive index is different for the two components. When light with an arbitrary SOP is incident on such a crystal, the two orthogonally polarized components are refracted at different angles. Each component goes through a Faraday rotator, which rotates the SOPs by 45°. The Faraday rotator is followed by a *half-wave plate*. The half-wave plate (a reciprocal device) rotates the SOPs by 45° in the clockwise direction for signals propagating from left to right, and by 45° in the counterclockwise direction for signals propagating from right to left. Therefore, the combination of the Faraday rotator and the half-wave plate converts the horizontal polarization into a vertical polarization and vice versa, and the two signals are combined by another SWP at the output. For reflected signals in the reverse direction, the half-wave plate and Faraday rotator cancel each other's effects, and the SOPs remain unchanged as they pass through these two devices and are thus not recombined by the SWP at the input.

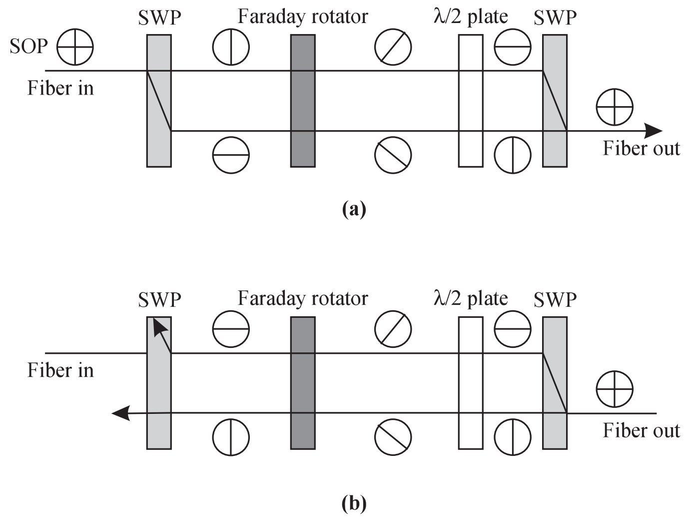

Figure 3.5 A polarization-independent isolator. The isolator is constructed along the same lines as a polarization-dependent isolator but uses spatial walk-off polarizers at the inputs and outputs. (a) Propagation from left to right. (b) Propagation from right to left.

3.3 *Multiplexers and Filters*

In this section, we will study the principles underlying the operation of a variety of wavelength selection technologies. Optical filters are essential components in transmission systems for at least two applications: to multiplex and demultiplex wavelengths in a WDM system—these devices are called multiplexers/demultiplexers—and to provide equalization of the gain and filtering of noise in optical amplifiers. Furthermore, understanding optical filtering is essential to understanding the operation of lasers later in this chapter.

The different applications of optical filters are shown in Figure 3.6. A simple filter is a two-port device that selects one wavelength and rejects all others. It may have an additional third port on which the rejected wavelengths can be obtained. A multiplexer combines signals at different wavelengths on its input ports onto a common output port, and a demultiplexer performs the opposite function. Multiplexers and demultiplexers are used in WDM terminals as well as in larger *wavelength crossconnects* and *wavelength add/drop multiplexers*.

Demultiplexers and multiplexers can be cascaded to realize *static* wavelength crossconnects (WXCs). In a static WXC, the crossconnect pattern is fixed at the time

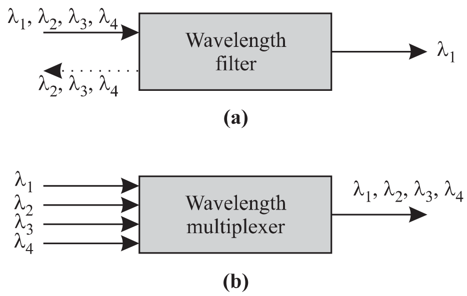

Figure 3.6 Different applications for optical filters in optical networks. (a) A simple filter, which selects one wavelength and either blocks the remaining wavelengths or makes them available on a third port. (b) A multiplexer, which combines multiple wavelengths into a single fiber. In the reverse direction, the same device acts as a demultiplexer to separate the different wavelengths.

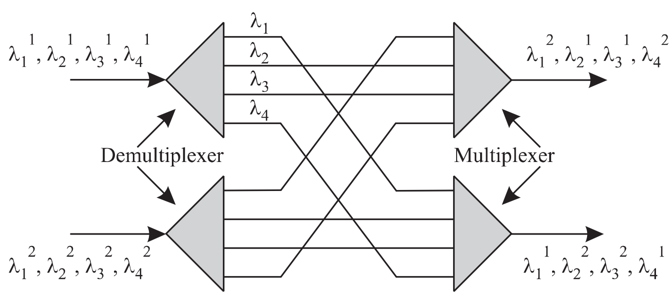

Figure 3.7 A static wavelength crossconnect. The device routes signals from an input port to an output port based on the wavelength.

the device is made and cannot be changed dynamically. Figure 3.7 shows an example of a static WXC. The device routes signals from an input port to an output port based on the wavelength. *Dynamic* WXCs can be constructed by combining optical switches with multiplexers and demultiplexers. Static WXCs are highly limited in terms of their functionality. For this reason, the devices of interest are dynamic rather than static WXCs. We will study different dynamic WXC architectures in Chapter 7.

A variety of optical filtering technologies are available. Their key characteristics for use in systems are the following:

1. Good optical filters should have low *insertion losses*. The insertion loss is the input-to-output loss of the filter.
2. The loss should be independent of the state of polarization of the input signals. The state of polarization varies randomly with time in most systems, and if the filter has a polarization-dependent loss, the output power will vary with time as well—an undesirable feature.
3. The passband of a filter should be insensitive to variations in ambient temperature. The *temperature coefficient* is measured by the amount of wavelength shift per unit degree change in temperature. The system requirement is that over the entire operating temperature range (about 100°C typically), the wavelength shift should be much less than the wavelength spacing between adjacent channels in a WDM system.
4. As more and more filters are cascaded in a WDM system, the passband becomes progressively narrower. To ensure reasonably broad passbands at the end of the cascade, the individual filters should have very flat passbands, so as to accommodate small changes in operating wavelengths of the lasers over time. This is measured by the 1 dB bandwidth, as shown in Figure 3.8.
5. At the same time, the passband skirts should be sharp to reduce the amount of energy passed through from adjacent channels. This energy is seen as *crosstalk* and degrades the system performance. The crosstalk suppression, or *isolation* of the filter, which is defined as the relative power passed through from the adjacent channels, is an important parameter as well.

In addition to all the performance parameters described, perhaps the most important consideration is cost. Technologies that require careful hand assembly tend to be more expensive. There are two ways of reducing the cost of optical filters. The first is to fabricate them using integrated-optic waveguide technology. This is analogous to semiconductor chips, although the state of integration achieved with optics is significantly less. These waveguides can be made on many substrates, including silica, silicon, InGaAs, and polymers. Waveguide devices tend to be inherently polarization dependent due to the geometry of the waveguides, and care must be taken to reduce the PDL in these devices. The second method is to realize all-fiber devices. Such devices are amenable to mass production and are inherently polarization independent. It is also easy to couple light in and out of these devices from/into other fibers. Both of these approaches are being pursued today.

All the filters and multiplexers we study use the property of *interference* among optical waves. In addition, some filters, for example, gratings, use the *diffraction* property—light from a source tends to spread in all directions depending on the

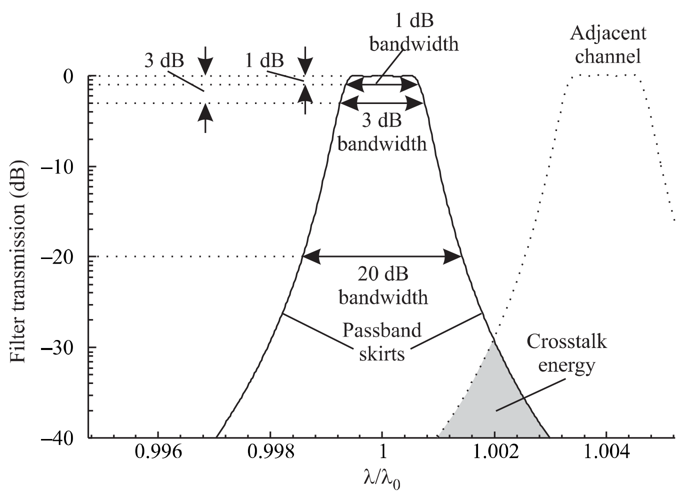

Figure 3.8 Characterization of some important spectral-shape parameters of optical filters. λ_0 is the center wavelength of the filter, and λ denotes the wavelength of the light signal.

incident wavelength. Table 3.1 compares the performance of different filtering technologies.

3.3.1 Gratings

The term *grating* is used to describe almost any device whose operation involves interference among multiple optical signals originating from the same source but with different relative *phase shifts*. An exception is a device where the multiple optical signals are generated by repeated traversals of a single cavity; such devices are called *etalons*. An electromagnetic wave (light) of angular frequency ω propagating, say, in the z direction has a dependence on z and t of the form $\cos(\omega t - \beta z)$. Here, β is the propagation constant and depends on the medium. The *phase* of the wave is $\omega t - \beta z$. Thus a relative phase shift between two waves from the same source can be achieved if they traverse two paths of different lengths.

Two examples of gratings are shown in Figure 3.9(a) and (b). Gratings have been widely used for centuries in optics to separate light into its constituent wavelengths. In WDM communication systems, gratings are used as demultiplexers to separate the individual wavelengths or as multiplexers to combine them. The Stimax grating of Table 3.1 is a grating of the type we describe in this section.

Table 3.1 Comparison of passive wavelength multiplexing/demultiplexing technologies. A 16-channel system with 100 GHz channel spacing is assumed. Other key considerations include center wavelength accuracy and manufacturability. All these approaches face problems in scaling with the number of wavelengths. TFMF is the dielectric thin-film multicavity filter, and AWG is the arrayed waveguide grating. For the fiber Bragg grating and the arrayed waveguide grating, the temperature coefficient can be reduced to 0.001 nm/°C by passive temperature compensation. The fiber Bragg grating is a single channel filter, and multiple filters need to be cascaded in series to demultiplex all 16 channels.

Filter Property	Fiber Bragg Grating	TFMF	AWG	Stimax Grating
1 dB BW (nm)	0.3	0.4	0.22	0.1
Isolation (dB)	25	25	25	30
Loss (dB)	0.2	7	5.5	6
PDL (dB)	0	0.2	0.5	0.1
Temp. coeff. (nm/°C)	0.01	0.0005	0.01	0.01

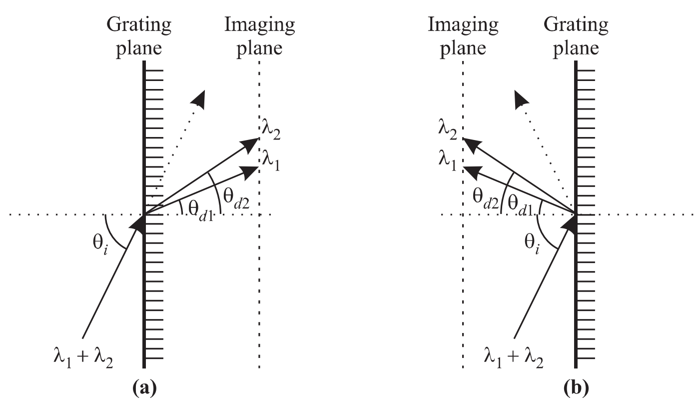

Figure 3.9 (a) A transmission grating and (b) a reflection grating. θ_i is the angle of incidence of the light signal. The angle at which the signal is diffracted depends on the wavelength ($\theta_d 1$ for wavelength λ_1 and $\theta_d 2$ for λ_2).

Consider the grating shown in Figure 3.9(a). Multiple narrow slits are spaced equally apart on a plane, called the *grating plane*. The spacing between two adjacent slits is called the *pitch* of the grating. Light incident from a source on one side of the grating is transmitted through these slits. Since each slit is narrow, by the phenomenon known as *diffraction,* the light transmitted through each slit spreads out in all directions. Thus each slit acts as a secondary source of light. Consider some other plane parallel to the grating plane at which the transmitted light from all the slits interferes. We will call this plane the *imaging plane*. Consider any point on this imaging plane. For wavelengths for which the individual interfering waves at this point are in phase, we have constructive interference and an enhancement of the light intensity at these wavelengths. For a large number of slits, which is the case usually encountered in practice, the interference is not constructive at other wavelengths, and there is little light intensity at this point from these wavelengths. Since different wavelengths interfere constructively at different points on the imaging plane, the grating effectively separates a WDM signal spatially into its constituent wavelengths. In a fiber optic system, optical fibers could be placed at different imaging points to collect light at the different wavelengths.

Note that if there were no diffraction, we would simply have light transmitted or reflected along the directed dotted lines in Figure 3.9(a) and (b). Thus the phenomenon of diffraction is key to the operation of these devices, and for this reason they are called *diffraction gratings*. Since multiple transmissions occur in the grating of Figure 3.9(a), this grating is called a *transmission grating*. If the transmission slits are replaced by narrow reflecting surfaces, with the rest of the grating surface being nonreflecting, we get the *reflection grating* of Figure 3.9(b). The principle of operation of this device is exactly analogous to that of the transmission grating. A majority of the gratings used in practice are reflection gratings since they are somewhat easier to fabricate. In addition to the plane geometry we have considered, gratings are fabricated in a concave geometry. In this case, the slits (for a transmission grating) are located on the arc of a circle. In many applications, a concave geometry leads to fewer auxiliary parts like lenses and mirrors needed to construct the overall device, say, a WDM demultiplexer, and is thus preferred.

The Stimax grating [LL84] is a reflection grating that is integrated with a concave mirror and the input and output fibers. Its characteristics are described in Table 3.1, and it has been used in commercially available WDM transmission systems. However, it is a bulk device that cannot be easily fabricated and is therefore relatively expensive. Attempts have been made to realize similar gratings in optical waveguide technology, but these devices are yet to achieve loss, PDL, and isolation comparable to the bulk version.

Principle of Operation

To understand quantitatively the principle of operation of a (transmission) grating, consider the light transmitted through adjacent slits as shown in Figure 3.10. The distance between adjacent slits—the *pitch* of the grating—is denoted by a. We assume that the light source is far enough away from the grating plane compared to a so that the light can be assumed to be incident at the same angle θ_i to the plane of the grating at each slit. We consider the light rays diffracted at an angle θ_d from the grating plane. The imaging plane, like the source, is assumed to be far away from the grating plane compared to the grating pitch. We also assume that the slits are small compared to the wavelength so that the phase change across a slit is negligible. Under these assumptions, it can be shown (Problem 3.4) that the path length difference between the rays traversing through adjacent slits is the difference in lengths between the line segments $\overline{AB}$ and $\overline{CD}$ and is given approximately by $a[\sin(\theta_i) - \sin(\theta_d)]$. Thus constructive interference at a wavelength λ occurs at the imaging plane among the rays diffracted at angle θ_d if the following *grating equation* is satisfied:

$$a[\sin(\theta_i) - \sin(\theta_d)] = m\lambda \tag{3.9}$$

for some integer m, called the *order* of the grating. The grating effects the separation of the individual wavelengths in a WDM signal since the grating equation is satisfied at different points in the imaging plane for different wavelengths. This is illustrated in Figure 3.9, where different wavelengths are shown being diffracted at the angles at which the grating equation is satisfied for that wavelength. For example, θ_{d1} is the angle at which the grating equation is satisfied for λ_1.

Note that the energy at a single wavelength is distributed over all the discrete angles that satisfy the grating equation (3.9) at this wavelength. When the grating is used as a demultiplexer in a WDM system, light is collected from only one of these angles, and the remaining energy in the other orders is lost. In fact, most of the energy will be concentrated in the zeroth-order ($m = 0$) interference maximum, which occurs at $\theta_i = \theta_d$ for all wavelengths. The light energy in this zeroth-order interference maximum is wasted since the wavelengths are not separated. Thus gratings must be designed so that the light energy is maximum at one of the other interference maxima. This is done using a technique called *blazing* [KF86, p. 386].

Figure 3.11 shows a blazed reflection grating with blaze angle α. In such a grating, the reflecting slits are inclined at an angle α to the grating plane. This has the effect of maximizing the light energy in the interference maximum whose order corresponds to the blazing angle. The grating equation for such a blazed grating can be derived as before; see Problem 3.5.

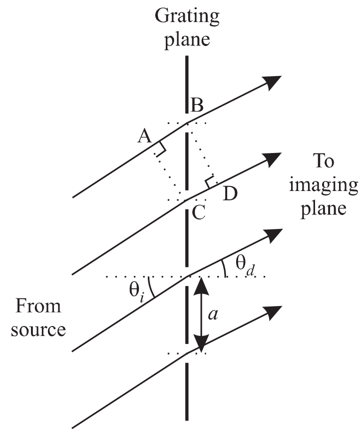

Figure 3.10 Principle of operation of a transmission grating. The reflection grating works in an analogous manner. The path length difference between rays diffracted at angle θ_d from adjacent slits is $\overline{AB} - \overline{CD} = a[\sin(\theta_i) - \sin(\theta_d)]$.

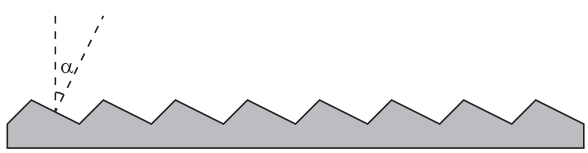

Figure 3.11 A blazed grating with blaze angle α. The energy in the interference maximum corresponding to the blaze angle is maximized.

3.3.2 Diffraction Pattern

So far, we have only considered the position of the diffraction maxima in the diffraction pattern. Often, we are also interested in the distribution of the intensity in the diffraction maxima. We can derive the distribution of the intensity by relaxing the assumption that the slits are much smaller than a wavelength, so that the phase change across a slit can no longer be neglected. Consider a slit of length w stretching from $y = -w/2$ to $y = w/2$. By reasoning along the same lines as we did in Figure 3.10, the light diffracted from position y at angle θ from this slit has a relative phase shift of $\phi(y) = (2\pi y \sin\theta)/\lambda$ compared to the light diffracted from $y = 0$. Thus, at the imaging plane, the amplitude $A(\theta)$ at angle θ is given by

$$\begin{aligned}\frac{A(\theta)}{A(0)} &= \frac{1}{w}\int_{-w/2}^{w/2} \exp(i\phi(y))\, dy \\ &= \frac{1}{w}\int_{-w/2}^{w/2} \exp(i2\pi(\sin\theta)y/\lambda)\, dy \\ &= \frac{\sin(\pi w \sin\theta/\lambda)}{\pi w \sin\theta/\lambda}. \end{aligned} \tag{3.10}$$

Observe that the amplitude distribution at the imaging plane is the Fourier transform of the rectangular slit. This result holds for a general diffracting aperture, and not just a rectangular slit. For this more general case, if the diffracting aperture or slit is described by $f(y)$, the amplitude distribution of the diffraction pattern is given by

$$A(\theta) = A(0)\int_{-\infty}^{\infty} f(y)\exp(2\pi i(\sin\theta)y/\lambda)\, dy. \tag{3.11}$$

The intensity distribution is given by $|A(\theta)|^2$. Here, we assume $f(y)$ is normalized so that $\int_{-\infty}^{\infty} f(y)\, dy = 1$. For a rectangular slit, $f(y) = 1/w$ for $|y| < w/2$ and $f(y) = 0$, otherwise, and the diffraction pattern is given by (3.10). For a pair of narrow slits spaced distance d apart,

$$f(y) = 0.5(\delta(y - d/2) + \delta(y + d/2))$$

and

$$A(\theta) = A(0)\cos(\pi(\sin\theta)\lambda/d).$$

The more general problem of N narrow slits is discussed in Problem 3.6.

3.3.3 Bragg Gratings

Bragg gratings are widely used in fiber optic communication systems. In general, any periodic perturbation in the propagating medium serves as a Bragg grating. This perturbation is usually a periodic variation of the refractive index of the medium. We will see in Section 3.5.1 that lasers use Bragg gratings to achieve single frequency operation. In this case, the Bragg gratings are "written" in waveguides. Bragg gratings written in fiber can be used to make a variety of devices such as filters, add/drop multiplexers, and dispersion compensators. We will see later that the Bragg grating principle also underlies the operation of the acousto-optic tunable filter. In this case, the Bragg grating is formed by the propagation of an acoustic wave in the medium.

Principle of Operation

Consider two waves propagating in opposite directions with propagation constants β_0 and β_1. Energy is coupled from one wave to the other if they satisfy the Bragg *phase-matching* condition

$$|\beta_0 - \beta_1| = \frac{2\pi}{\Lambda},$$

where Λ is the period of the grating. In a Bragg grating, energy from the forward propagating mode of a wave at the right wavelength is coupled into a backward propagating mode. Consider a light wave with propagation constant β_1 propagating from left to right. The energy from this wave is coupled onto a scattered wave traveling in the opposite direction at the same wavelength provided

$$|\beta_0 - (-\beta_0)| = 2\beta_0 = \frac{2\pi}{\Lambda}.$$

Letting $\beta_0 = 2\pi n_{\text{eff}}/\lambda_0$, λ_0 being the wavelength of the incident wave and n_{eff} the effective refractive index of the waveguide or fiber, the wave is reflected provided

$$\lambda_0 = 2n_{\text{eff}}\Lambda.$$

This wavelength λ_0 is called the Bragg wavelength. In practice, the reflection efficiency decreases as the wavelength of the incident wave is detuned from the Bragg wavelength; this is plotted in Figure 3.12(a). Thus if several wavelengths are transmitted into a fiber Bragg grating, the Bragg wavelength is reflected while the other wavelengths are transmitted.

The operation of the Bragg grating can be understood by reference to Figure 3.13, which shows a periodic variation in refractive index. The incident wave is reflected from each period of the grating. These reflections add in phase when the path length in wavelength λ_0 each period is equal to half the incident wavelength λ_0. This is equivalent to $n_{\text{eff}}\Lambda = \lambda_0/2$, which is the Bragg condition.

The reflection spectrum shown in Figure 3.12(a) is for a grating with a uniform refractive index pattern change across its length. In order to eliminate the undesirable side lobes, it is possible to obtain an *apodized* grating, where the refractive index change is made smaller toward the edges of the grating. (The term *apodized* means "to cut off the feet.") The reflection spectrum of an apodized grating is shown in Figure 3.12(b). Note that, for the apodized grating, the side lobes have been drastically reduced but at the expense of increasing the main lobe width.

The index distribution across the length of a Bragg grating is analogous to the grating aperture discussed in Section 3.3.2, and the reflection spectrum is obtained as the Fourier transform of the index distribution. The side lobes in the case of a uniform refractive index profile arise due to the abrupt start and end of the grating,

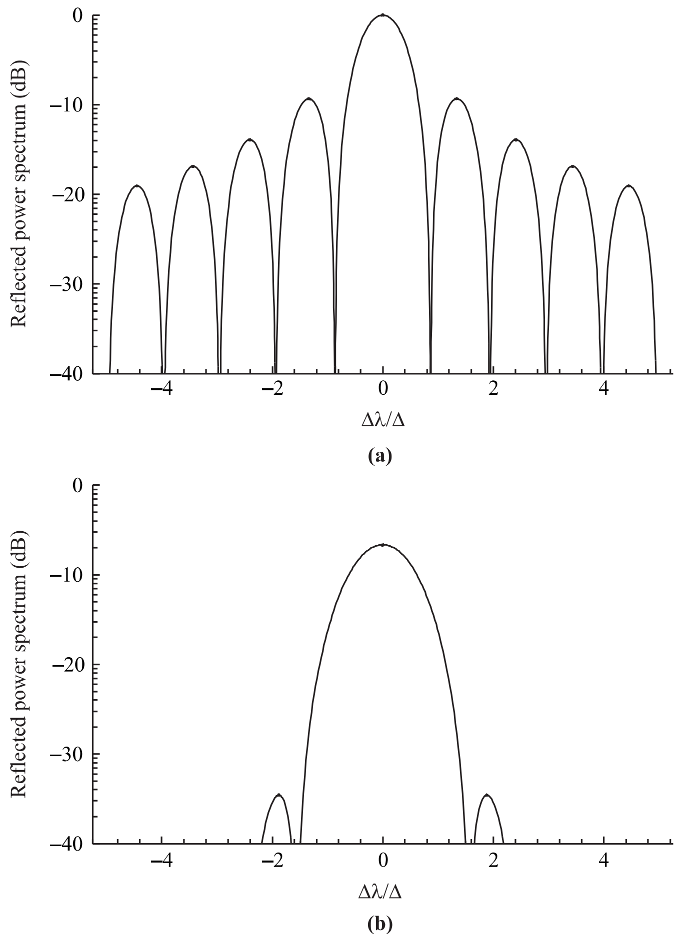

Figure 3.12 Reflection spectra of Bragg gratings with (a) uniform index profile and (b) apodized index profile. Δ is a measure of the bandwidth of the grating and is the wavelength separation between the peak wavelength and the first reflection minimum, in the uniform index profile case. Δ is inversely proportional to the length of the grating. $\Delta\lambda$ is the detuning from the phase-matching wavelength.

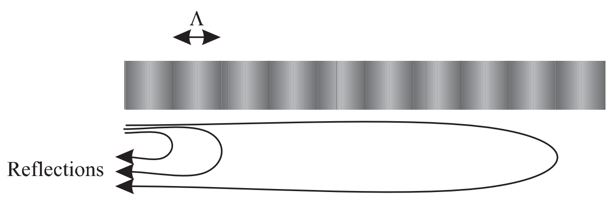

Figure 3.13 Principle of operation of a Bragg grating.

which result in a sinc(.) behavior for the side lobes. Apodization can be achieved by gradually starting and ending the grating. This technique is similar to pulse shaping used in digital communication systems to reduce the side lobes in the transmitted spectrum of the signal.

The bandwidth of the grating, which can be measured, for example, by the width of the main lobe, is inversely proportional to the length of the grating. Typically, the grating is a few millimeters long in order to achieve a bandwidth of 1 nm.

3.3.4 *Fiber Gratings*

Fiber gratings are attractive devices that can be used for a variety of applications, including filtering, add/drop functions, and compensating for accumulated dispersion in the system. Being all-fiber devices, their main advantages are their low loss, ease of coupling (with other fibers), polarization insensitivity, low temperature-coefficient, and simple packaging. As a result, they can be extremely low-cost devices.

Gratings are written in fibers by making use of the *photosensitivity* of certain types of optical fibers. A conventional silica fiber doped with germanium becomes extremely photosensitive. Exposing this fiber to ultraviolet (UV) light causes changes in the refractive index within the fiber core. A grating can be written in such a fiber by exposing its core to two interfering UV beams. This causes the radiation intensity to vary periodically along the length of the fiber. Where the intensity is high, the refractive index is increased; where it is low, the refractive index is unchanged. The change in refractive index needed to obtain gratings is quite small—around 10^{-4}. Other techniques, such as *phase masks,* can also be used to produce gratings. A phase mask is a diffractive optical element. When it is illuminated by a light beam, it splits the beams into different diffractive orders, which then interfere with one another to write the grating into the fiber.

Fiber gratings are classified as either *short-period* or *long-period* gratings, based on the period of the grating. Short-period gratings are also called Bragg gratings and have periods that are comparable to the wavelength, typically around 0.5 μm. We discussed the behavior of Bragg gratings in Section 3.3.3. Long-period gratings, on the other hand, have periods that are much greater than the wavelength, ranging from a few hundred micrometers to a few millimeters.

Fiber Bragg Gratings

Fiber Bragg gratings can be fabricated with extremely low loss (0.1 dB), high wavelength accuracy ($\pm$ 0.05 nm is easily achieved), high adjacent channel crosstalk suppression (40 dB), as well as flat tops.

The temperature coefficient of a fiber Bragg grating is typically 1.25×10^{-2} nm/°C due to the variation in fiber length with temperature. However, it is possible to compensate for this change by packaging the grating with a material that has a negative thermal expansion coefficient. These passively temperature-compensated gratings have temperature coefficients of around 0.07×10^{-2} nm/°C. This implies a very small 0.07 nm center wavelength shift over an operating temperature range of 100°C, which means that they can be operated without any active temperature control.

These properties of fiber Bragg gratings make them very useful devices for system applications. Fiber Bragg gratings are finding a variety of uses in WDM systems, ranging from filters and optical add/drop elements to dispersion compensators. A simple optical drop element based on fiber Bragg gratings is shown in Figure 3.14(a). It consists of a three-port circulator with a fiber Bragg grating. The circulator transmits light coming in on port 1 out on port 2 and transmits light coming in on port 2 out on port 3. In this case, the grating reflects the desired wavelength λ_2, which is then dropped at port 3. The remaining three wavelengths are passed through. It is possible to implement an add/drop function along the same lines, by introducing a coupler to add the same wavelength that was dropped, as shown in Figure 3.14(b). Many variations of this simple add/drop element can be realized by using gratings in combination with couplers and circulators. A major concern in these designs is that the reflection of these gratings is not perfect, and as a result, some power at the selected wavelength leaks through the grating. This can cause undesirable crosstalk, and we will study this effect in Chapter 5.

Fiber Bragg gratings can also be used to compensate for dispersion accumulated along the link. We will study this application in Chapter 5 in the context of dispersion compensation.

Long-Period Fiber Gratings

Long-period fiber gratings are fabricated in the same manner as fiber Bragg gratings and are used today primarily as filters inside erbium-doped fiber amplifiers to compensate for their nonflat gain spectrum. As we will see, these devices serve as very

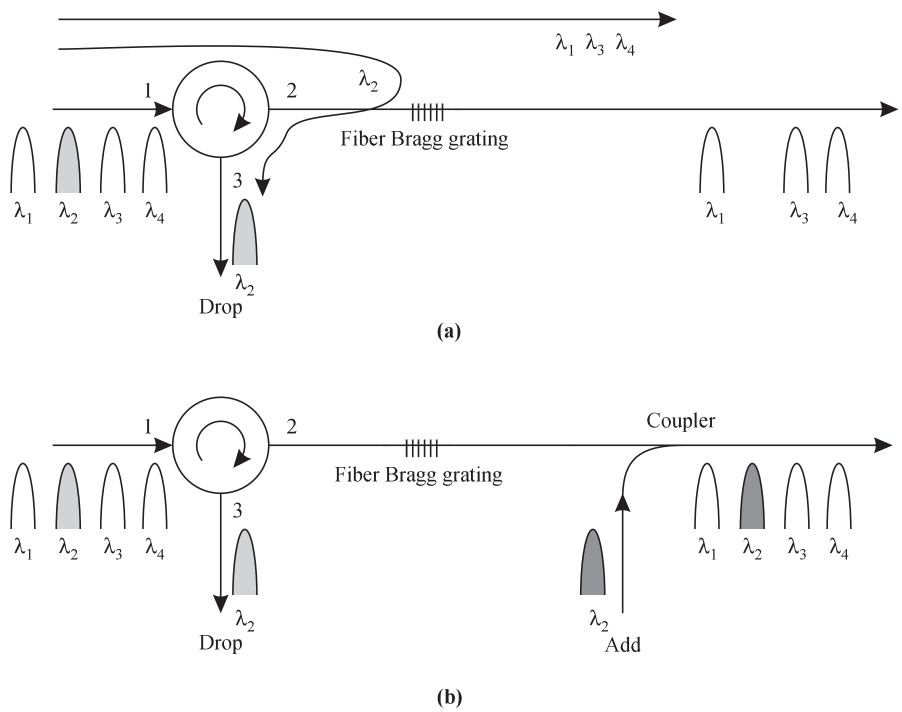

Figure 3.14 Optical add/drop elements based on fiber Bragg gratings. (a) A drop element. (b) A combined add/drop element.

efficient band rejection filters and can be tailored to provide almost exact equalization of the erbium gain spectrum. Figure 3.15 shows the transmission spectrum of such a grating. These gratings retain all the attractive properties of fiber gratings and are expected to become widely used for several filtering applications.

Principle of Operation

These gratings operate on somewhat different principles than Bragg gratings. In fiber Bragg gratings, energy from the forward propagating mode in the fiber core at the right wavelength is coupled into a backward propagating mode. In long-period gratings, energy is coupled from the forward propagating mode in the fiber core onto other forward propagating modes in the cladding. These cladding modes are extremely lossy, and their energy decays rapidly as they propagate along the fiber, due to losses at the cladding–air interface and due to microbends in the fiber. There are many cladding modes, and coupling occurs between a core mode at a given

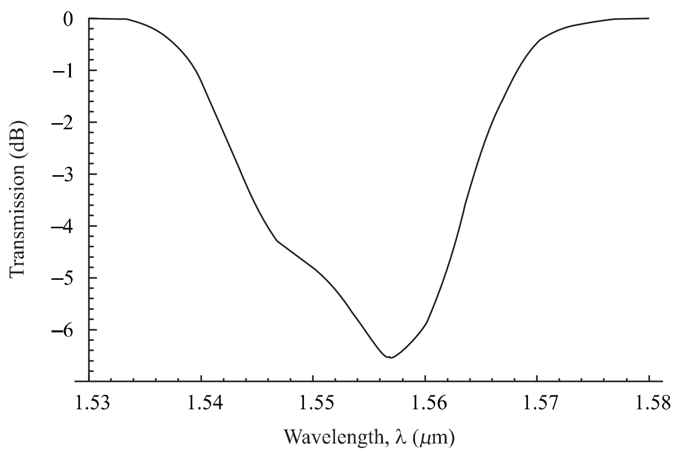

Figure 3.15 Transmission spectrum of a long-period fiber Bragg grating used as a gain equalizer for erbium-doped fiber amplifiers. (After [Ven96a].)

wavelength and a cladding mode depending on the pitch of the grating Λ, as follows: if β denotes the propagation constant of the mode in the core (assuming a single-mode fiber) and β_{cl}^{p} that of the pth-order cladding mode, then the phase-matching condition dictates that

$$\beta - \beta_{\text{cl}}^{p} = \frac{2\pi}{\Lambda}.$$

In general, the difference in propagation constants between the core mode and any one of the cladding modes is quite small, leading to a fairly large value of Λ in order for coupling to occur. This value is usually a few hundred micrometers. (Note that in Bragg gratings the difference in propagation constants between the forward and backward propagating modes is quite large, leading to a small value for Λ, typically around 0.5 μm.) If n_{eff} and n_{eff}^{p} denote the effective refractive indices of the core and pth-order cladding modes, then the wavelength at which energy is coupled from the core mode to the cladding mode can be obtained as

$$\lambda = \Lambda(n_{\text{eff}} - n_{\text{eff}}^{p}),$$

where we have used the relation $\beta = 2\pi n_{\text{eff}}/\lambda$.

Therefore, once we know the effective indices of the core and cladding modes, we can design the grating with a suitable value of Λ so as to cause coupling of energy out of a desired wavelength band. This causes the grating to act as a wavelength-dependent loss element. Methods for calculating the propagation

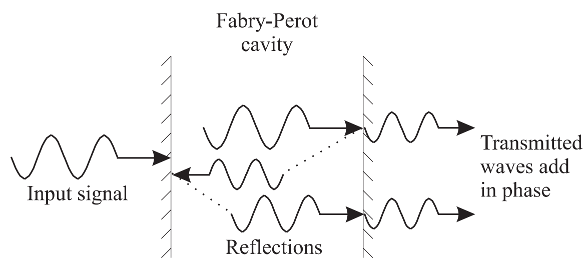

Figure 3.16 Principle of operation of a Fabry-Perot filter.

constants for the cladding modes are discussed in [Ven96b]. The amount of wavelength-dependent loss can be controlled during fabrication by controlling the UV exposure time. Complicated transmission spectra can be obtained by cascading multiple gratings with different center wavelengths and different exposures. The example shown in Figure 3.15 was obtained by cascading two such gratings [Ven96a]. These gratings are typically a few centimeters long.

3.3.5 Fabry-Perot Filters

A Fabry-Perot filter consists of the cavity formed by two highly reflective mirrors placed parallel to each other, as shown in Figure 3.16. This filter is also called a Fabry-Perot interferometer or etalon. The input light beam to the filter enters the first mirror at right angles to its surface. The output of the filter is the light beam leaving the second mirror.

This is a classical device that has been used widely in interferometric applications. Fabry-Perot filters have been used for WDM applications in several optical network testbeds. There are better filters today, such as the thin-film resonant multicavity filter that we will study in Section 3.3.6. These latter filters can be viewed as Fabry-Perot filters with wavelength-dependent mirror reflectivities. Thus the fundamental principle of operation of these filters is the same as that of the Fabry-Perot filter. The Fabry-Perot cavity is also used in lasers (see Section 3.5.1).

Compact Fabry-Perot filters are commercially available components. Their main advantage over some of the other devices is that they can be tuned to select different channels in a WDM system, as discussed later.

Principle of Operation

The principle of operation of the device is illustrated in Figure 3.16. The input signal is incident on the left surface of the cavity. After one pass through the cavity, as