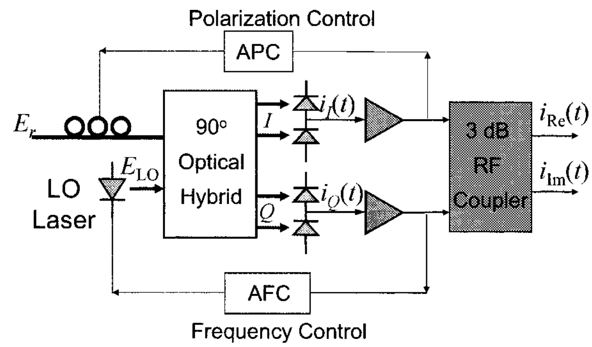

Figure 3.5. A heterodyne image-rejection receiver.

Figure 3.5 shows an image-rejection receiver with an optical front-end similar to the quadrature receiver of Fig. 3.4 with an 90° optical hybrid and two balanced receivers. The image-rejection receiver of Fig. 3.5 is for heterodyne receiver with $\omega_{\mathrm{IF}} > 0$. Assume that there are two signals at ω_{c1} and ω_{c2} with a relationship of

$$\omega_{\mathrm{IF}} = \omega_{c1} - \omega_{\mathrm{LO}} = \omega_{\mathrm{LO}} - \omega_{c2}. \tag{3.53}$$

If the two signals are

$$E_{c1}(t) = A_{s1}(t)e^{j\omega_{c1}t + j\phi_{s1}(t)}, \text{ and } E_{c2}(t) = A_{s2}(t)e^{j\omega_{c2}t + j\phi_{s2}(t)}, \tag{3.54}$$

similar to Eqs. (3.49) and (3.50) and without going into details, we obtain two photocurrents of

$$\begin{aligned} i_I(t) &= RA_{s1}(t)A_L \cos[(\omega_{c1} - \omega_{\mathrm{LO}})t + \phi_{s1}(t)] \\ &\quad + RA_{s2}(t)A_L \cos[(\omega_{\mathrm{LO}} - \omega_{c2})t - \phi_{s2}(t)], \end{aligned} \tag{3.55}$$

and

$$\begin{aligned} i_Q(t) &= RA_{s1}(t)A_L \sin[(\omega_{c1} - \omega_{\mathrm{LO}})t + \phi_{s1}(t)] \\ &\quad - RA_{s2}(t)A_L \sin[(\omega_{\mathrm{LO}} - \omega_{c2})t - \phi_{s2}(t)]. \end{aligned} \tag{3.56}$$

After a 90° microwave hybrid[2], we obtain

$$i_{\mathrm{Re}}(t) = \sqrt{2}RA_{s1}(t)A_L \cos[(\omega_{c1} - \omega_{\mathrm{LO}})t + \phi_{s1}(t)], \tag{3.57}$$

$$i_{\mathrm{Im}}(t) = \sqrt{2}RA_{s2}(t)A_L \cos[(\omega_{\mathrm{LO}} - \omega_{c2})t - \phi_{s2}(t)] \tag{3.58}$$

[2]The transfer matrix must be

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix}$$

with a 90° shift from Eq. (3.1). The transfer matrix is called 90° microwave hybrid but similar matrix is for 180° optical hybrid.

for the real and image frequency band, respectively.

In an image-rejection heterodyne receiver of Fig. 3.5, even without an optical filter to filter-out the image-band amplified spontaneous emission, the amplifier-noise limited SNR is $\rho_s = N_s/n_{\text{eq}}$ from Eq. (3.40). Not only reject the signal from the image band, the image-rejection receiver also rejects the noise from the image band.

In another application of the image-rejection receiver of Fig. 3.5, the LO laser can have a frequency between two adjacent WDM channels. The real and image bands can be processed to receive two channels having larger or smaller frequency than the LO laser.

Used by Chikama et al. (1990a) and Lachs et al. (1990) for WDM systems, heterodyne image-rejection receiver was proposed by Darcie and Glance (1986), Glance (1986b), and Westphah and Strebel (1988). Even without optical filter, as indicated in Glance et al. (1988), Walker et al. (1990) and Jørgensen et al. (1992) showed that the SNR is improved by 3 dB using image-rejection receiver. The output SNR of image-rejection receiver is the same as that of homodyne receiver.

1.5 SNR of Basic Coherent Receivers

Table 3.1 summaries the SNR of system with different receiver structures. Single-branch receiver is more likely to be limited by the noise from LO laser.

For system limited by amplifier noise, without optical filter, heterodyne receiver is 3-dB worse than homodyne receiver. Image-rejection receiver eliminates the effects of the amplifier noise from the image frequency band even for the system without optical filtering. Homodyne and heterodyne receivers have the same SNR for both cases of having image-rejection or optical filtering. The optical filter of a heterodyne receiver must have a bandwidth conformed to the relationship of Eq. (3.12).

For system limited by shot-noise, the performance of heterodyne receiver is always 3-dB worse than homodyne receiver. This 3-dB difference was given in all standard textbooks (Agrawal, 2002, Betti et al., 1995, Okoshi and Kikuchi, 1988).

In later parts of this book, system performance is analyzed based on the representation of a received signal by

$$r(t) = A_s(t)\cos[\omega_{\text{IF}} + \phi_s(t)] + n(t) \tag{3.59}$$

with the SNR from Table 3.1. The noise in the receiver is considered to be within the narrow receiver bandwidth of the receiver and with a band-pass representation of

$$n(t) = n_1(t)\cos\omega_{\text{IF}}t - n_2(t)\sin\omega_{\text{IF}}t \tag{3.60}$$

Table 3.1. Comparison of the SNR of Different Receiver Structures.

Receiver Types	Limited Noise Sources	
	Shot Noise	Amplifier Noise
Single-branch heterodyne w/ optical filter†	ηN_s	$\dfrac{N_s}{n_{\mathrm{eq}}}$
Single-branch heterodyne w/o optical filter†	ηN_s	$\dfrac{N_s}{2n_{\mathrm{eq}}}$
Single-branch homodyne†	$2\eta N_s$	$\dfrac{N_s}{n_{\mathrm{eq}}}$
Balanced received heterodyne w/ optical filter	ηN_s	$\dfrac{N_s}{n_{\mathrm{eq}}}$
Balanced received heterodyne w/o optical filter	ηN_s	$\dfrac{N_s}{2n_{\mathrm{eq}}}$
Balanced received homodyne	$2\eta N_s$	$\dfrac{N_s}{n_{\mathrm{eq}}}$
Quadrature homodyne	$2\eta N_s$	$\dfrac{N_s}{n_{\mathrm{eq}}}$
Image-rejection heterodyne receiver	ηN_s	$\dfrac{N_s}{n_{\mathrm{eq}}}$

†Single-branch receiver is more likely to be limited by LO noise.

with

$$E\{n^2(t)\} = E\{n_1^2(t)\} = E\{n_2^2(t)\} = \sigma_n^2. \tag{3.61}$$

As discussed earlier, a quantum-limited system has the limit of a quantum efficiency of $\eta = 1$ or equivalent spontaneous emission of $n_{\mathrm{eq}} = 1$. The quantum-limited SNR is equal to the average number of photons per bit (or per symbol) for heterodyne system limited by either shot or amplifier noise.

2. Performance of Synchronous Receivers

When an optical PLL is used to track the phase of the LO laser for homodyne systems or an electrical PLL is used to track the phase of an IF oscillator for heterodyne systems, the system is called a coherent system according to the terminology of conventional digital communications (Proakis, 2000). In coherent optical communications, system with phase tracking is called synchronous detection system. This section calculates the error probability of synchronous detection systems as a function of SNR. With the same SNR of ρ_s, homodyne and heterodyne systems have the same performance. This section considers only heterodyne systems with balanced receiver.

2.1 Amplitude-Shift Keying

When the optical carrier is ASK modulated, the signal current of Eq. (3.24) in a heterodyne receiver can be expressed as

$$s_1(t) = A\cos\omega_{\text{IF}}t, \tag{3.62}$$

$$s_2(t) = 0. \tag{3.63}$$

The above binary ASK signal can be received by the heterodyne receiver of Fig. 1.3(b).

Including noise, the overall received signal is

$$r(t) = s(t) + n(t) = \begin{cases} [A + n_1(t)]\cos\omega_{\text{IF}}t - n_2(t)\sin\omega_{\text{IF}}t & \text{for } s_1(t) \\ n_1(t)\cos\omega_{\text{IF}}t - n_2(t)\sin\omega_{\text{IF}}t & \text{for } s_2(t) \end{cases}. \tag{3.64}$$

At the output of an ASK receiver with PLL, the decision variable is $r_d(t) = A + n_1(t)$ and $n_1(t)$ for $s_1(t)$ and $s_2(t)$, respectively. With decision threshold of $\frac{1}{2}A$, the error probability is

$$p_e = \frac{1}{2}\int_{-\infty}^{\frac{A}{2}} p_1(x)\mathrm{d}x + \frac{1}{2}\int_{\frac{A}{2}}^{+\infty} p_2(x)\mathrm{d}x, \tag{3.65}$$

where

$$p_1(x) = \frac{1}{\sqrt{2\pi}\sigma_n}e^{-\frac{(x-A)^2}{2\sigma_n^2}}, \tag{3.66}$$

$$p_2(x) = \frac{1}{\sqrt{2\pi}\sigma_n}e^{-\frac{x^2}{2\sigma_n^2}} \tag{3.67}$$

are the Gaussian probability density function (p.d.f.) of the decision random variables.

We obtain

$$p_e = \frac{1}{2}\text{erfc}\left(\frac{A}{2\sqrt{2}\sigma_n}\right) \tag{3.68}$$

$$= \frac{1}{2}\text{erfc}\left(\sqrt{\frac{\rho_s}{2}}\right), \tag{3.69}$$

where the SNR is , $\text{erfc}(x) = 2/\sqrt{\pi}\int_x^{\infty} e^{-t^2}\mathrm{d}t$ is the complementary error function, and

$$\rho_s = \frac{A^2/4}{\sigma_n^2}. \tag{3.70}$$

with signal power equal to $\frac{1}{2}\frac{A^2}{2} + 0 = \frac{A^2}{4}$.

To achieve an error probability of 10^{-9}, a SNR of $\rho_s = 36$ (15.6 dB) is required. The quantum-limited binary ASK signal requires 36 photons/bit.

2.2 Phase-Shift Keying

When the light is PSK modulated, the signal of Eq. (3.24) in a heterodyne receiver can be expressed as

$$s_1(t) = A \cos \omega_{\mathrm{IF}} t, \tag{3.71}$$

$$s_2(t) = -A \cos \omega_{\mathrm{IF}} t. \tag{3.72}$$

The above binary PSK signal can be received by the heterodyne receiver of Fig. 1.3(b). Including noise, the overall received signal is

$$r(t) = s(t)+n(t) = \begin{cases} [A + n_1(t)] \cos \omega_{\mathrm{IF}} t - n_2(t) \sin \omega_{\mathrm{IF}} t & \text{for } s_1(t) \\ [-A + n_1(t)] \cos \omega_{\mathrm{IF}} t - n_2(t) \sin \omega_{\mathrm{IF}} t & \text{for } s_2(t) \end{cases} \tag{3.73}$$

At the output of the PLL, the decision random variable is $r_d(t) = \pm A + n_1(t)$. With a decision threshold of zero, the error probability is

$$p_e = \frac{1}{2} \int_{-\infty}^{0} p_1(x) \mathrm{d}x + \frac{1}{2} \int_{0}^{+\infty} p_2(x) \mathrm{d}x, \tag{3.74}$$

where

$$p_1(x) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(x-A)^2}{2\sigma_n^2}}, \tag{3.75}$$

$$p_2(x) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(x+A)^2}{2\sigma_n^2}} \tag{3.76}$$

are the Gaussian p.d.f. of the decision random random variables.

We obtain

$$p_e = \frac{1}{2} \mathrm{erfc} \left(\frac{A}{\sqrt{2}\sigma_n} \right) \tag{3.77}$$

$$= \frac{1}{2} \mathrm{erfc} \left(\sqrt{\rho_s} \right), \tag{3.78}$$

where the SNR is

$$\rho_s = \frac{A^2/2}{\sigma_n^2}. \tag{3.79}$$

To achieve an error probability of 10^{-9}, a SNR of $\rho_s = 18$ (12.5 dB) is required for an improvement of 3-dB over ASK signal. The quantum-limited binary PSK signal requires 18 photons/bit.

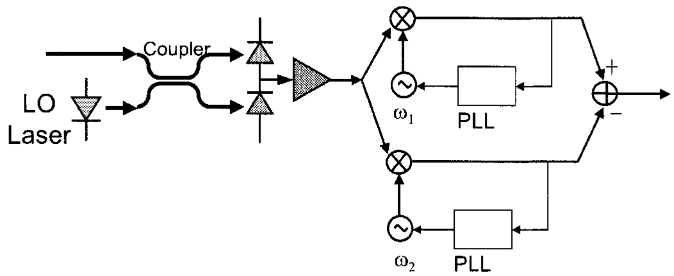

Figure 3.6. A heterodyne synchronous receiver for FSK signal.

Synchronous receivers had been demonstrated mostly for PSK signal for its superior performance (Kahn et al., 1990, Kazovsky and Atlas, 1990, Kazovsky et al., 1990, Norimatsu et al., 1990, Schöpflin et al., 1990, Watanabe et al., 1989). There were other experiments to transmit some reference carriers without phase locking (Cheng and Okoshi, 1989, Wandernoth, 1992, Watanabe et al., 1992). There is recently interest to conduct PSK experiment (Cho et al., 2004c, Taylor, 2004).

2.3 Frequency-Shift Keying

When the optical carrier is frequency-shift keying (FSK) modulated, the signal of a heterodyne receiver can be expressed as

$$s_1(t) = A\cos\omega_1 t, \tag{3.80}$$

$$s_2(t) = A\cos\omega_2 t, \tag{3.81}$$

where ω_1 and ω_2 are two angular frequencies with orthogonal condition of

$$\int_0^T s_1(t)s_2(t)\mathrm{d}t = 0, \tag{3.82}$$

where T is the symbol interval. The two frequencies should be separated by $\omega_1 - \omega_2 = k\pi/T$, $k = \pm1, \pm2, \ldots$, for the orthogonal condition. In a synchronous heterodyne FSK receiver of Fig. 3.6, two electrical PLL are required to phase lock the two oscillators with frequencies of either ω_1 or ω_2 for the two FSK signals. Equivalently, two matched filters are used with filter response matching to $s_1(t)$ and $s_2(t)$, respectively. The difference of the two outputs of Fig. 3.6 decides whether $s_1(t)$ or $s_2(t)$ is transmitted.

With the orthogonal condition, the overall received signal including noise is

$$r(t) = s(t) + n(t) = \begin{cases} [A + n_{11}(t)]\cos\omega_1 t - n_{12}(t)\sin\omega_1 t & \text{for } s_1(t) \\ [A + n_{21}(t)]\cos\omega_2 t - n_{22}(t)\sin\omega_2 t & \text{for } s_2(t) \end{cases}. \tag{3.83}$$

where $n_{11}(t)$, $n_{21}(t)$, $n_{12}(t)$, and $n_{22}(t)$ are independent of each other with the same variance of σ_n^2. When $s_1(t)$ is transmitted, the correct decision is $A + n_{11}(t) > n_{21}(t)$ or $A + n_{11}(t) - n_{21}(t) > 0$. The noise of $n_{11}(t) - n_{21}(t)$ has a variance of $2\sigma_n^2$. The error probability is the same as that of PSK signal but with a noise variance of $2\sigma_n^2$, we obtain

$$p_e = \frac{1}{2}\text{erfc}\left(\frac{A}{2\sigma_n}\right) \tag{3.84}$$

$$= \frac{1}{2}\text{erfc}\left(\sqrt{\frac{\rho_s}{2}}\right), \tag{3.85}$$

where the SNR is

$$\rho_s = \frac{A^2/2}{\sigma_n^2}. \tag{3.86}$$

A FSK signal has the same performance as ASK signal. The same as ASK signal, to achieve an error probability of 10^{-9}, a SNR of $\rho_s = 36$ (15.6 dB) is required. The quantum-limited binary FSK signal requires 36 photons/bit.

The performance of synchronous receiver, the same as digital signal with a matched filter provided by phase tracking, is analyzed in Proakis (2000) or the early paper of Yamamoto (1980).

3. Performance of Asynchronous Receivers

All of ASK, DPSK, and FSK signals can be detected without phase tracking. The detection is based on the comparison of the power or envelope of the signal. This type of detection is called noncoherent detection in conventional digital communications (Proakis, 2000) and asynchronous receiver in coherent optical communications. In this section, we consider asynchronous heterodyne receiver with signal processing by electrical circuits in the IF.

3.1 Envelope Detection of Heterodyne ASK Signal

Figure 3.7 shows the receiver for envelope detection of heterodyne binary ASK signal. The signal first passes through a band-pass filter (BPF) to limit the amount of noise. After the BPF, the signal is the

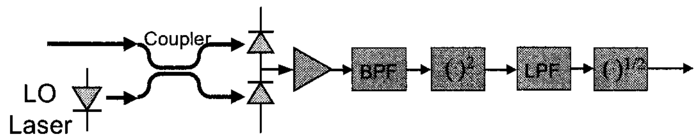

Figure 3.7. A heterodyne asynchronous receiver for ASK signal.

same as that of Eq. (3.64). The signal of Eq. (3.64) is squared, low-pass filtered (LPF), square-rooted, to obtain the envelope of

$$r_d(t) = \begin{cases} \sqrt{[A + n_1(t)]^2 + n_2^2(t)} & \text{for } s_1(t) \\ \sqrt{n_1^2(t) + n_2^2(t)} & \text{for } s_2(t) \end{cases} \tag{3.87}$$

with p.d.f. of

$$p_1(r) = \frac{r}{\sigma_n^2} I_0\left(\frac{rA}{\sigma_n^2}\right) \exp\left(-\frac{r^2 + A^2}{2\sigma_n^2}\right), \tag{3.88}$$

$$p_2(r) = \frac{r}{\sigma_n^2} \exp\left(-\frac{r^2}{2\sigma_n^2}\right), \tag{3.89}$$

as Rice and Rayleigh distribution, respectively, where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. In the decision random variable of Eq. (3.87), some constant factors related to gain and loss of the squarer, LPF, and the square-root components are ignored. The envelope of Eq. (3.87) is the same as the amplitude of the signal. The error probability of the signal is

$$p_e = \frac{1}{2}\left[1 - \int_{r_{\text{th}}}^{\infty} p_1(r) \mathrm{d}r\right] + \frac{1}{2}\int_{r_{\text{th}}}^{\infty} p_2(r) \mathrm{d}r, \tag{3.90}$$

where r_{th} is the threshold. Using the Marcum's Q-function defined by Eq. (3.A.3) from Appendix 3.A, we obtain

$$p_e = \frac{1}{2}\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{r_{\text{th}}}{\sigma_n}\right)\right] + \frac{1}{2}\exp\left(-\frac{r_{\text{th}}^2}{2\sigma_n^2}\right). \tag{3.91}$$

The optimal threshold can be found by $dp_e/dr_{\text{th}} = 0$ as

$$I_0\left(\frac{Ar_{\text{th}}}{\sigma_n^2}\right) \exp\left(-\frac{A^2}{2\sigma_n^2}\right) = 1. \tag{3.92}$$

The optimal decision threshold is difficult to find analytically, we may use the approximation of $r_{\text{th}} = A/2$. With this threshold, the second

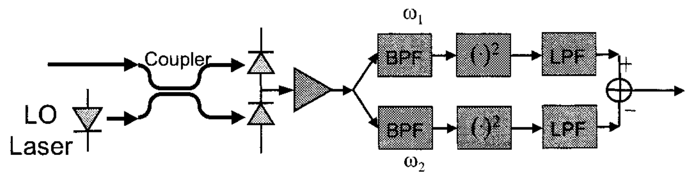

Figure 3.8. A heterodyne asynchronous receiver for FSK signal.

term of Eq. (3.91) is larger than the first term and the error probability is approximately equal to

$$\begin{aligned} p_e &\approx \frac{1}{2}\exp\left(-\frac{A^2}{8\sigma_n^2}\right) \\ &= \frac{1}{2}\exp\left(-\frac{\rho_s}{2}\right). \end{aligned} \tag{3.93}$$

Based on the approximation of Eq. (3.93), the required SNR for an error probability of 10^{-9} is $\rho_s = 40$ (16 dB). The quantum limit is 40 photons/bit. The asynchronous receiver is about 0.4 dB worse than the synchronous receiver in Sec. 3.2.1.

3.2 Dual-Filter Detection of FSK Signal

Figure 3.8 shows an asynchronous heterodyne receiver for FSK signal based on two BPF matched to the FSK signals of $s_1(t)$ and $s_2(t)$ with center angular frequencies of ω_1 and ω_2, respectively. Based on Eq. (3.83), when $s_1(t)$ is transmitted, the received signal at the first filter centered at ω_1 is the same as that of Eq. (3.87). Although not necessary, a square root is assumed for the signal of Fig. 3.8 for convenience. When $s_1(t)$ is transmitted, we obtain

$$r_1(t) = \sqrt{[A + n_{11}(t)]^2 + n_{12}^2(t)}, \tag{3.94}$$

and

$$r_2(t) = \sqrt{n_{21}^2(t) + n_{22}^2(t)} \tag{3.95}$$

with p.d.f. of

$$p_1(r_1) = \frac{r_1}{\sigma_n^2} I_0\left(\frac{r_1 A}{\sigma_n^2}\right)\exp\left(-\frac{r_1^2 + A^2}{2\sigma_n^2}\right), \tag{3.96}$$

$$p_2(r_2) = \frac{r_2}{\sigma_n^2}\exp\left(-\frac{r_2^2}{2\sigma_n^2}\right) \tag{3.97}$$

as Rice and Rayleigh distribution, respectively.

Bit error occurs when $r_1 < r_2$, or

$$p_e = \Pr\{r_1 < r_2\} = \Pr\{r_1^2 < r_2^2\}, \tag{3.98}$$

because both $r_1 \geq 0$ and $r_2 \geq 0$. From Eq. (3.98), the square-root component in the receiver of Fig. 3.8 is optional. The error probability is

$$\begin{aligned} p_e &= \int_0^\infty p_1(r_1) \int_{r_1}^\infty p_2(r_2) \mathrm{d}r_2 \mathrm{d}r_1 \\ &= \int_0^\infty \frac{r_1}{\sigma_n^2} I_0\left(\frac{r_1 A}{\sigma_n^2}\right) \exp\left(-\frac{2r_1^2 + A^2}{2\sigma_n^2}\right) \mathrm{d}r_1 \\ &= \frac{1}{2} \exp\left(-\frac{A^2}{4\sigma_n^2}\right) \int_0^\infty \frac{2r_1}{\sigma_n^2} I_0\left(\frac{2r_1 A}{2\sigma_n^2}\right) \exp\left(-\frac{4r_1^2 + A^2}{4\sigma_n^2}\right) \mathrm{d}r_1 \\ &= \frac{1}{2} \exp\left(-\frac{A^2}{4\sigma_n^2}\right) \\ &= \frac{1}{2} \exp\left(-\frac{\rho_s}{2}\right). \end{aligned} \tag{3.99}$$

The above error probability is valid only if the two signals are orthogonal. The performance of FSK signal is similar to that of ASK signal The required SNR for an error probability of 10^{-9} is $\rho_s = 40$ (16 dB). The quantum limit is 40 photons/bit.

A FSK signal can also be detected based on a single filter. The signal after the filter is the same as an ASK signal. The performance of FSK signal with a single filter is the same as that of ASK signal with an error probability of $p_e = \frac{1}{2} \exp(-\rho_s/4)$. While an ASK signal has no power at the "0" level, the power of FSK signal is the same at both "0" and "1" levels. The performance of single-filter detected FSK signal is 3-dB worse than the equivalent ASK signal. Intuitively, using a single filter, the FSK signal is also 3-dB worse than a dual-filter receiver.

Single filter FSK experiment was conducted by Emura et al. (1984) and Park et al. (1990), and dual-filter FSK experiment was conducted by Emura et al. (1990b).

3.3 Heterodyne Differential Detection of DPSK Signal

In another representation for a heterodyne DPSK system, the received signal is

$$r(t) = \Re\left\{\left[Ae^{j\phi_s(t)} + n(t)\right] e^{j\omega_{\mathrm{IF}} t}\right\}, \tag{3.100}$$

or

$$r(t) = \Re\left\{\tilde{r}(t) e^{j\omega_{\mathrm{IF}} t}\right\}, \tag{3.101}$$

where $\tilde{r}(t) = Ae^{j\phi_s(t)} + n(t)$ is the baseband representation of the IF signal. The DPSK signal is demodulated by delay-and-multiplier circuits of Fig. 1.4(b). If $\tilde{r}(t)$ is expressed by the polar representation of $\tilde{r}(t) = \tilde{A}_1(t)e^{j\tilde{\phi}_1(t)}$ with $\tilde{\phi}_1(t)$ include the noisy phase from the contribution of $n(t)$, the demodulated signal after a low-pass filter is

$$\begin{aligned} r_d(t) &= \tilde{A}_1(t)\tilde{A}_1(t-T)\cos\left[\omega_{\mathrm{IF}}\tau + \tilde{\phi}_1(t) - \tilde{\phi}_1(t-T)\right] \\ &= \Re\left\{\tilde{r}(t)\tilde{r}^*(t-T)\right\}. \end{aligned} \tag{3.102}$$

In Eq. (3.102), two symbols of τ and T are used to represent the delay of the delay-and-multiplier circuits. The delay of τ must be approximated equal to T and $\omega_{\mathrm{IF}}\tau$ must be an integer multiple of 2π for a noiseless decision variable of $r_d(t) = \pm A^2$ with $\phi_s(t) - \phi_s(t-T) = 0, \pi$, respectively. For a large frequency of ω_{IF}, a small variation of τ around the time interval T may give $\omega_{\mathrm{IF}}\tau$ equal to an integer multiple of 2π. Later in this book assumes that $\tau = T$ and $\omega_{\mathrm{IF}}T$ is equal to an integer multiple of 2π at the same time.

Some simple algebra gives

$$r_d(t) = \left|\frac{\tilde{r}(t) + \tilde{r}(t-T)}{2}\right|^2 - \left|\frac{\tilde{r}(t) - \tilde{r}(t-T)}{2}\right|^2. \tag{3.103}$$

The signals of $\tilde{r}(t) + \tilde{r}(t-T)$ and $\tilde{r}(t) - \tilde{r}(t-T)$ are independent of each other. With $r_1 = |\tilde{r}(t) + \tilde{r}(t-T)|/2$ and $r_2 = |\tilde{r}(t) + \tilde{r}(t-T)|/2$, assume that $\phi_s(t) = \phi_s(t-T)$, the error probability is similar to Eq. (3.98) with

$$p_e = \Pr\{r_1 < r_2\} = \Pr\{r_1^2 < r_2^2\} = \Pr\{r_d < 0\}. \tag{3.104}$$

If $\phi_s(t) = \phi_s(t-T)$, $r_1^2 = |A + n(t)/2 + n(t-T)/2|^2$ as the first term of Eq. (3.103) is the square of a Gaussian random variable with mean of A and variance of σ_n^2 and $r_2^2 = |n(t) - n(t-T)|^2/4$ as the second term of Eq. (3.103) is the square of a Gaussian random variable with zero mean and variance of σ_n^2. The error probability is the same as that for FSK signal of Eq. (3.85) with equivalent SNR of A^2/σ_n^2, we obtain

$$\begin{aligned} p_e &= \frac{1}{2}\exp\left(-\frac{A^2}{2\sigma_n^2}\right) \\ &= \frac{1}{2}\exp\left(-\rho_s\right). \end{aligned} \tag{3.105}$$

The required SNR for an error probability of 10^{-9} is $\rho_s = 20$ (13 dB) for DPSK signal. The quantum limit is 20 photons/bit. DPSK signal is about 3-dB better than ASK signal.

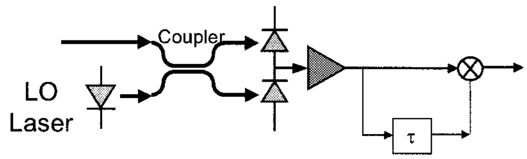

Figure 3.9. A heterodyne asynchronous receiver for CPFSK signal based on delay-and-multiplier circuits.

The error probability of Eq. (3.105) was derived by Cahn (1959) using approximation. DPSK signal was analyzed the same as orthogonal or FSK signal in Stein (1964). Using two time intervals, the performance of DPSK signal is 3-dB better than that of FSK signal. In Eq. (3.103), although the mean of the signal is the same as that of the FSK signal, the noise variance is reduced by half because the noise is the average over two time intervals.

Heterodyne DPSK signal was demonstrated in Chikama et al. (1988), Creaner et al. (1988), Meissner (1989), Naito et al. (1990), and Gnauck et al. (1990).

3.4 Heterodyne Receiver for CPFSK Signal

In binary continuous-phase frequency-shift keying (CPFSK) transmission, the signal in each time interval is equal to

$$s(t) = A\cos\left[\omega_{\mathrm{IF}}t \pm \pi\Delta f t + \phi_0\right], \tag{3.106}$$

where ϕ_0 depends the phase of previous symbols to ensure continuous phase operation and Δf is the frequency deviation between the "0" and "1" states. With a received signal of $r(t) = s(t) + n(t)$, we may define $\tilde{r}(t) = Ae^{\pm j\pi\Delta f t + j\phi_0} + n(t) = \tilde{A}_1(t)e^{j\tilde{\phi}_1(t)}$ similar to that for DPSK signal. The CPFSK signal can be demodulated using the asynchronous receiver of Fig. 3.9 based on the delay-and-multiplier circuits similar to that of Fig. 1.4(b) for DPSK signals. In the CPFSK receiver of Fig. 3.9, the delay is τ instead of the one-bit delay of T in Fig. 1.4(b). Similar to the case of DPSK signal of Eq. (3.103), the decision random variable is

$$\begin{aligned} r_d &= \tilde{A}_1(t)\tilde{A}_1(t-\tau)\cos\left[\omega_{\mathrm{IF}}\tau + \tilde{\phi}_1(t) - \tilde{\phi}_1(t-\tau)\right] \\ &= \Re\left\{\tilde{r}(t)\tilde{r}^*(t-\tau)e^{j\omega_{\mathrm{IF}}\tau}\right\}, \end{aligned} \tag{3.107}$$

where $\tilde{\phi}_1(t) = \pm\pi\Delta f t + \phi_0 + \phi_n(t)$ with $\phi_n(t)$ as the noisy phase from $n(t)$. The differential phase is $\tilde{\phi}_1(t) - \tilde{\phi}_1(t-\tau) = \pm\pi\Delta f\tau + \phi_n(t) - \phi_n(t-\tau)$. The receiver achieves its optimal performance for a design of $\omega_{\mathrm{IF}}\tau =$

$k\pi + \frac{1}{2}\pi$ with k as an integer. Excluding noise, the decision variable is $r_d(t) = \pm A^2 \sin(\Delta f \pi \tau)$. For the best system efficiency, $r_d(t) = \pm A^2$ if $\Delta f \tau = 1/2$.

With the design of $\Delta f \tau = 1/2$, assume that the phase of noise of $\phi_n(t)$ and $\phi_n(t - \tau)$ are independent of each other, the performance of CPFSK signal with differential detection is the same as DPSK signal with error probability of

$$p_e = \frac{1}{2} \exp\left(-\rho_s\right). \tag{3.108}$$

The most interesting case is $\Delta f = 1/2T$ when $\tau = T$. The frequency separation of $\Delta f = 1/2T$ is the minimum frequency separation for two orthogonal frequencies of Eq. (3.82). The CPFSK signal with $\Delta f = 1/2T$ is called minimum shift keying (MSK) modulation. Similar to DPSK signal, MSK signal may be demodulated using a one-bit delay and multiplier. The performance of MSK signal is also the same as DPSK signal. From Proakis (2000), MSK signal has more compact spectrum than DPSK signal.

Because CPFSK signal can be generated directly modulated a semiconductor laser from Sec. 2.4, it was the most popular scheme for coherent optical communications (Emura et al., 1990a, Iwashita and Matsumoto, 1987, Iwashita and Takachio, 1988, Park et al., 1991, Takachio et al., 1989).

3.5 Frequency Discriminator for FSK Signal

FSK signal can also be detected asynchronously by frequency discriminator of Fig. 3.10. The frequency discriminator may be the most popular receiver for analog frequency modulation (FM) signal for its simplicity. The system must have high SNR to ensure the correct operation of the frequency discriminator.

The frequency discriminator of Fig. 3.10 consists of two band-pass filters having frequency responses of

$$H_1(f) = \begin{cases} 2\pi K(f - f_{\mathrm{IF}}) & f_{\mathrm{IF}} < f < f + \Delta f/2 \\ 0 & \text{otherwise} \end{cases} \tag{3.109}$$

and

$$H_2(f) = \begin{cases} 2\pi K(f_{\mathrm{IF}} - f) & f - \Delta f/2 < f < f_{\mathrm{IF}} \\ 0 & \text{otherwise} \end{cases}, \tag{3.110}$$

where K is the slope of the frequency discriminator and Δf is the bandwidth of the frequency discriminator and the frequency separation of

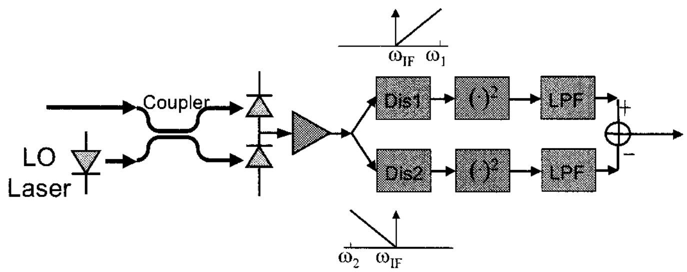

Figure 3.10. A heterodyne asynchronous receiver for FSK signal using frequency discriminator.

two FSK signals of $2\pi\Delta f = \omega_1 - \omega_2$. In time domain, the operation of the frequency discriminator is equivalent to a linear operation of $K\frac{d}{dt}$.

Assume that $s_1(t)$ is transmitted with a signal before the discriminator as

$$A\cos\omega_1 t + n_1(t)\cos\omega_{\mathrm{IF}}t - n_2(t)\sin\omega_{\mathrm{IF}}t, \tag{3.111}$$

the output of $H_1(f)$ is

$$r_1(t) = AK(\omega_1 - \omega_{\mathrm{IF}})\sin\omega_1 t + K\frac{dn_{11}(t)}{dt}\cos(\omega_{\mathrm{IF}}t + \pi\Delta f t/2) - K\frac{dn_{12}(t)}{dt}\sin(\omega_{\mathrm{IF}}t + \pi\Delta f t/2), \tag{3.112}$$

where $\omega_1 - \omega_{\mathrm{IF}} = \pi\Delta f$, $n_{11}(t)$ and $n_{12}(t)$ are the part of $n_1(t)$ and $n_2(t)$ in the frequency between f_{IF} and $f_{\mathrm{IF}} + \Delta f/2$. The output of $H_2(f)$ is

$$r_2(t) = K\frac{dn_{21}(t)}{dt}\cos(\omega_{\mathrm{IF}}t + \pi\Delta f t/2) - K\frac{dn_{22}(t)}{dt}\sin(\omega_{\mathrm{IF}}t + \pi\Delta f t/2). \tag{3.113}$$

where $n_{21}(t)$ and $n_{22}(t)$ are the part of $n_1(t)$ in the frequency smaller than the IF of f_{IF}. The signal of $r_1(t)$ is similar to an amplitude-modulated signal with a power of $\pi^2[AK\Delta f]^2/2$. With the derivative operation as a linear filter, the noise of $Kdn_{11}(t)/dt$ has a variance of

$$K^2 E\left\{\left|\frac{dn_{11}}{dt}\right|^2\right\} = (2\pi K)^2\int_0^{B_d} N_n f^2 \mathrm{d}f = (2\pi K)^2\frac{1}{3}B_d^3 N_n = \frac{4}{3}\pi^2 K^2 B_d^2\sigma_n^2. \tag{3.114}$$

where N_n is the spectral density of $n_{11}(t)$ and $\sigma_n^2 = N_0 B_d$. At the output of the frequency discriminator, the signal is similar to that case of orthogonal signals demonstrated by two BPF followed by envelope detection.

The equivalent SNR of the discriminator output signal is

$$\rho_{\mathrm{eq}} = \frac{3\rho_s}{8}\left(\frac{\Delta f}{B_d}\right)^2. \tag{3.115}$$

Similar to the error probability of Eq. (3.99), we obtain

$$p_e = \frac{1}{2}\exp\left[-\frac{3\rho_s}{16}\left(\frac{\Delta f}{B_d}\right)^2\right]. \tag{3.116}$$

For the case that $\Delta f = 2B_d$, we obtain $p_e = \frac{1}{2}\exp(-3\rho_s/4)$, 1.76 dB improvement over the asynchronous receiver of Eq. (3.99) but twice the spectral bandwidth. Using the method based on frequency discriminator, the system performance is improved with the frequency expansion of $\Delta f/B_d$. Similar to analog FM, detection based on frequency discriminator expands the signal bandwidth to obtain improved system performance. However, the error probability of Eq. (3.116) should be considered as an approximation. While the noise outside B_d in Eq. (3.114) is ignored in the deviation, those noise gives noise-to-noise beating thought the squarer of Fig. 3.10 and degrades the performance of the system, especially at low SNR.

3.6 Envelope Detection of Correlated Binary Signals

FSK, DPSK, and MSK signals are special cases of signal modulation formats based on two orthogonal signals. The dual-filter detection of FSK signal and differential detection of both DPSK and MSK signals are asynchronous or noncoherent detection of two orthogonal signals. For envelope detection of binary signals, the receiver is the same as that of Fig. 3.8 with the band-pass filters representing two matched filters. If the two signals are correlated with a correlation coefficient of $|\rho|$, the p.d.f. of the outputs of two matched filters are

$$p_1(r_1) = \frac{r_1}{\sigma_n^2} I_0\left(\frac{r_1 A}{\sigma_n^2}\right)\exp\left(-\frac{r_1^2 + A^2}{2\sigma_n^2}\right), \tag{3.117}$$

$$p_2(r_2) = \frac{r_2}{\sigma_n^2} I_0\left(\frac{r_2 |\rho| A}{\sigma_n^2}\right)\exp\left(-\frac{r_2^2 + |\rho|^2 A^2}{2\sigma_n^2}\right). \tag{3.118}$$

The random variables of R_1 and R_2 are correlated with each other. In order to derive the error probability of $p_e = \Pr\{R_2 > R_1\}$, a transform is required such that $R_2^2 - R_1^2 = \Gamma_2^2 - \Gamma_1^2$ in which the random variables of

Γ_2 and Γ_1 are independent of each other and also have Rice distribution. While the amplitude parameter of R_1 and R_2 are A and $|\rho|A$, respectively, the amplitude parameters of Γ_1 and Γ_2 are $A\left[1 \pm \sqrt{1-|\rho|^2}\right]^{1/2}$, respectively. From Appendix 3.A, the error probability of correlated binary signal with envelope detection is

$$p_e = Q(a,b) - \frac{1}{2}e^{-(a^2+b^2)/2} I_0(ab), \tag{3.119}$$

where

$$a = \sqrt{\frac{\rho_s}{2}\left(1 - \sqrt{1-|\rho|^2}\right)},$$
$$b = \sqrt{\frac{\rho_s}{2}\left(1 + \sqrt{1-|\rho|^2}\right)}. \tag{3.120}$$

For orthogonal signal, $|\rho| = 0$, $a = 0$, and $b = \sqrt{\rho_s}$, $p_e = Q(0,b) - \frac{1}{2}e^{-b^2/2} = \frac{1}{2}e^{-b^2/2} = \frac{1}{2}e^{-\rho_s/2}$, the same as the error probability of Eq. (3.85).

The error probability of Eq. (3.119) was first derived by Helstrom (1955) based on direct integration. The method to find the error probability here is based on Stein (1964) and Schwartz et al. (1966). Proakis (2000, Appendix B) also derived the error probability of Eq. (3.119). While the error probability of Eq. (3.119) is not useful if the two binary signals are well-designed without correlation, further degradation that induces correlation can be analyzed based on Eq. (3.119).

4. Performance of Direct-Detection Receivers

Other than the intensity-modulation/direct-detection (IMDD) systems of Fig. 1.1, both DPSK and FSK signals can be directly detected without mixing with an LO laser. DPSK and FSK signals can be detected using interferometer or optical filter. Direct-detection receiver is simpler than both homodyne and heterodyne receivers that require the mixing with an LO laser. This section analyzes the performance of typical direct-detection receivers for ASK or on-off keying, DPSK, and FSK signals.

4.1 Intensity-Modulation/Direct-Detection Receiver

IMDD systems of Fig. 1.1 are the simplest optical communication schemes to converse information with the presence or absence of light. The receiver is just a photodiode that converts the optical intensity to photocurrent. For system limited by amplifier noises, the performance is analyzed in, for example, Agrawal (2002), Desurvire (1994), and Becker et al. (1999).

Quantum Limit for Systems without Amplifiers

In a system without optical amplifiers, due to the quantum nature of photons, the number of photons has a Poisson distribution. At the on-state, the probability of having k photons is

$$p_k = \frac{N_b^k}{k!} e^{-N_b}, \tag{3.121}$$

where N_b is the number of photons in the on state of a binary on-off keying signal. If the detection is based on the presence or absence of photons, the error probability is

$$p_e = \frac{1}{2} p_0 = \frac{1}{2} e^{-N_b} \tag{3.122}$$

with no photon, where the factor of 1/2 is the probability of the on state. With no photon to send, there is no error for the off state. The average number of photons is $N_s = N_b/2$. In order to achieve an error probability of 10^{-9}, an average of 10 photons/bit are required. If the quantum efficiency of η is less than unity, the required number of photons increases by the factor of η^{-1}.

Practical on-off keying receivers require thousands of photons per bit, mostly due to the contribution from the thermal noise at the receiver circuitry. As an example, assume a thermal noise density of $i_{\text{th}} = 5$ pA/$\sqrt{\text{Hz}}$, corresponding to a receiver sensitivity of -25.2 and -28.2 dBm for 10- and 2.5-Gb/s systems, respectively, if shot noise is ignored and a photodiode responsivity of $R = 1$ is assumed. For systems operating in 1.55 μm, the number of photons per bit is 4.7×10^4 and 2.3×10^4 for 10- and 2.5-Gb/s systems for an error probability of 10^{-9}. Without the usage of optical amplification, practical receiver always requires the number of photons per bit many order larger than the quantum limit of 10 photons/bit, even for receiver with very good sensitivity.

Amplifier-Noise Limited System

If the on-off keying system is limited by amplifier noise, the received electric field is

$$E_r(r) = [A_s(t) + n_x(t)]e^{j\omega_c t}\mathbf{x} + n_y(t)e^{j\omega_c t}\mathbf{y}, \tag{3.123}$$

where the transmitted data are contained in the amplitude of $A_s(t) \in \{0, A\}$ for on-off keying signal. In the direct-detection receiver, the above electric field is converted by a photodiode to photocurrent of

$$i(t) = R|A_s(t) + n_x(t)|^2 + R|n_y(t)|^2, \tag{3.124}$$

where shot noise is ignored by assuming that the amplifier noise is the major degradation. If the noise from orthogonal polarization of $n_y(t)$ is ignored, the system is the same as that using envelope detection for heterodyne ASK receiver in Sec. 3.3.1.

The common factor of photodiode responsivity of $R = 1$ is assumed in Eq. (3.124) without loss of generality. Further assumed an optical matched filter preceding the photodiode, in the on state with $A_s(t) = A$, we obtain

$$r_{\text{on}}(t) = [A + n_{x1}(t)]^2 + n_{x2}^2(t) + n_{y1}^2(t) + n_{y2}^2(t) \tag{3.125}$$

with p.d.f. of

$$p_1(y) = \frac{1}{2\sigma_n^2}\sqrt{\frac{y}{A^2}}e^{-(A^2+y)/2\sigma_n^2} I_1\left(\sqrt{y}\frac{A}{\sigma_n^2}\right), \qquad y \geq 0 \tag{3.126}$$

as the noncentral chi-square (χ^2) p.d.f. with four degrees of freedom.

In the off state with $A_s(t) = 0$, we obtain

$$r_{\text{off}}(t) = n_{x1}^2(t) + n_{x2}^2(t) + n_{y1}^2(t) + n_{y2}^2(t) \tag{3.127}$$

with p.d.f. of

$$p_2(y) = \frac{1}{4\sigma_n^4} y e^{-y/2\sigma_n^2}, \qquad y \geq 0 \tag{3.128}$$

as the χ^2 p.d.f. with four degrees of freedom, also called Gamma distribution.

With a threshold of y_{th}, the error probability of the system is

$$\begin{aligned} p_e &= \frac{1}{2}\int_0^{y_{\text{th}}} p_1(y)\mathrm{d}y + \frac{1}{2}\int_{y_{\text{th}}}^{\infty} p_2(y)\mathrm{d}y \\ &= \frac{1}{2}\left[1 - Q_2\left(\frac{A}{\sigma_n}, \frac{\sqrt{y_{\text{th}}}}{\sigma_n}\right)\right] + \frac{1}{2}\exp\left(-\frac{y_{\text{th}}}{2\sigma_n^2}\right)\left(1 + \frac{y_{\text{th}}}{2\sigma_n^2}\right), \end{aligned} \tag{3.129}$$

where

$$Q_2(a, b) = Q(a, b) + \frac{b}{a}e^{-(a^2+b^2)/2} I_1(ab) \tag{3.130}$$

is the second-order generalized Marcum Q function.

The optimal threshold can be determined by

$$\sqrt{y_{\text{th}}}\frac{A}{2\sigma_n^2} = e^{-A^2/2\sigma_n^2} I_1\left(\sqrt{y_{\text{th}}}\frac{A}{\sigma_n^2}\right), \qquad y \geq 0. \tag{3.131}$$

In a simplified analysis, we can approximate the decision threshold as $y_{\rm th} = A^2/4$, the error probability is approximately equal to

$$\begin{aligned} p_e &\approx \frac{1}{2}\int_{y_{\rm th}}^{\infty} p_2(y)\mathrm{d}y \\ &= \frac{1}{2}\exp\left(-\frac{A^2}{8\sigma_n^2}\right)\left(1+\frac{A^2}{8\sigma_n^2}\right) \\ &= \frac{1}{2}\exp\left(-\frac{\rho_s}{2}\right)\left(1+\frac{\rho_s}{2}\right). \end{aligned} \tag{3.132}$$

With the inclusion of the amplified noise from orthogonal polarization of $n_y(t)$, the error probability is increased by a factor of $1+\rho_s/2$. Using a direct-detection receiver, based on the error probability of Eq. (3.132), a SNR of $\rho_s = 46.4$ (16.7 dB) is required to achieve an error probability of 10^{-9}. The quantum-limited receiver requires 46.4 photons/bit. The inclusion of the noise from orthogonal polarization degrades the receiver by about 0.7 dB.

Error Probability Based on Gaussian Approximation

The error probability can be analyzed based on Gaussian approximation with the assumption of two received signals of

$$r_{\rm on}(t) = I_1 + n_1(t), \tag{3.133}$$

$$r_{\rm off}(t) = I_0 + n_0(t), \tag{3.134}$$

where $n_0(t)$ and $n_1(t)$ are assumed zero-mean Gaussian noise with variances of $E\{n_1^2(t)\} = \sigma_1^2$ and $E\{n_0^2(t)\} = \sigma_0^2$, respectively, I_1 and I_0 are the mean photocurrents for the on and off states. In general, $I_1 = RP_r$ and $I_0 = 0$.

The p.d.f. of the on and off states are

$$p_1(x) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-(x-I_1)^2/2\sigma_1^2}, \tag{3.135}$$

$$p_0(x) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(x-I_0)^2/2\sigma_0^2}. \tag{3.136}$$

The optimal decision threshold can be determined by

$$p_1(x_{\rm th}) = p_0(x_{\rm th}), \qquad I_0 < x_{\rm th} < I_1 \tag{3.137}$$

of

$$x_{\rm th} = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}. \tag{3.138}$$

In the derivation of the optimal threshold, we assume that $\sigma_0 \neq \sigma_1$ but $\log(\sigma_1/\sigma_0) = 0$ as an approximation. Without the approximation of $\log(\sigma_1/\sigma_0) = 0$, the optimal threshold is a difficult to calculate but without providing further accuracy.

Defined a Q-factor of

$$Q = \frac{I_1 - x_{\rm th}}{\sigma_1} = \frac{x_{\rm th} - I_0}{\sigma_0} = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}, \tag{3.139}$$

the error probability is equal to

$$p_e = \frac{1}{2}\mathrm{erfc}\left(\frac{Q}{\sqrt{2}}\right). \tag{3.140}$$

With the Gaussian approximation, the noise variances are equal to

$$\sigma_1^2 = \sigma^2_{A_s - n_s} + \sigma^2_{n_s - n_s} + \sigma^2_{\rm sh} + \sigma^2_{\rm th}, \tag{3.141}$$

$$\sigma_0^2 = \sigma^2_{n_s - n_s} + \sigma^2_{\rm sh} + \sigma^2_{\rm th}, \tag{3.142}$$

where

$$\sigma^2_{A_s - n_s} = 4R^2 P_r S_{\rm sps} B_d, \tag{3.143}$$

$$\sigma^2_{n_s - n_s} = 4R^2 S^2_{\rm sps} \Delta\nu_{\rm opt} B_d, \tag{3.144}$$

$$\sigma^2_{\rm sh} = 2eR(P_r + 2S_{\rm sps}\Delta\nu_{\rm opt})B_d, \tag{3.145}$$

$$\sigma^2_{\rm th} = \frac{4kT}{R_L}. \tag{3.146}$$

Thermal noise is included in the above equations for the case that the received signal is very small or for system without amplifier noises. A direct-detection on-off keying system is potentially limited by thermal instead of amplifier noise.

For the specific case of the signals of Eqs. (3.125) and (3.127) with amplifier noise from orthogonal polarization, we obtain

$$I_0 = 0, \qquad I_1 = A^2, \tag{3.147}$$

$$\sigma_0^2 = 8\sigma_n^4, \qquad \sigma_1^2 = 4\sigma_n^2 A^2 + 8\sigma_n^4, \tag{3.148}$$

and

$$\begin{aligned} Q &= \frac{A^2}{2\sqrt{2}\sigma_n^2 + 2\sigma_n\sqrt{A^2 + 2\sigma_n^2}} \\ &= \frac{\sqrt{2}\rho_s}{1 + \sqrt{2\rho_s + 1}}. \end{aligned} \tag{3.149}$$

Based on the Gaussian approximation, the required SNR to achieve an error probability of 10^{-9} is $\rho_s = 36 + 6\sqrt{2} = 44.5$ (16.5 dB).

When the optical filter preceding the receiver has a wide bandwidth, direct-detection on-off keying system can be approximately analyzed by adding identical and independent noise terms to both Eqs. (3.125) and (3.127) (Humblet and Azizoğlu, 1991, Marcuse, 1990, 1991). The number of noise terms is equal to twice the ratio of the optical bandwidth to data rates, the factor of two taking into account the two polarizations of amplifier noise. While Marcuse (1990, 1991) and Humblet and Azizoğlu (1991) sum independent and identical random variables to model amplifier noises, Lee and Shim (1994) sums independent random variables with variance depending on the combined effects of electrical and optical filter responses. The optical filter may also model further linear effects of chromatic and polarization-mode dispersion. These two methods are widely used for performance evaluation in direct-detection on-off keying systems (Bosco et al., 2001, Chan and Conradi, 1997, Forestieri, 2000, Holzlohner et al., 2002, Roudas et al., 2002).

Comparison of Different Models

Figure 3.11 shows the error probability of ASK signal detected using various types of receiver that are also analyzed based on different assumptions. The error probability of Eq. (3.69) with synchronous receiver has the lowest error probability for the same SNR. The error probability of Eq. (3.93) for envelope detection is also shown for comparison together as the error probability calculated with the optimal threshold of Eq. (3.92) as dashed-lines. The error probability of Eq. (3.132) for direct-detection is shown as solid line with the corresponding error probability calculated with the optimal threshold of Eq. (3.131) as dashed-lines. The error probability with optimal threshold of Eq. (3.131) almost overlaps with Eq. (3.93). The Gaussian approximation of Eq. (3.140) using Q factor is shown in Fig. 3.11 as dotted-line.

From Fig. 3.11, the performance of ASK signals with envelope detection can be evaluated using Eq. (3.93). Compared with the error probability with optimal threshold, the approximation of Eq. (3.93) is just about 0.1 dB worse at the error probability of 10^{-9}.

For direct-detection receiver, the Gaussian approximation overestimates the error probability and gives an approximated SNR penalty of about 0.45 dB comparing with the one with optimal threshold. Because the Gaussian approximation also uses an optimal threshold according to its own model, the performance with Gaussian approximation is actually better than the error probability with sub-optimal threshold of Eq. (3.132) by 0.2 dB. In practical system, for either direct- or envelope-detection, the error probability of Eq. (3.93) can be used.