or frequency-modulated signals have direct-detection receiver based on interferometer or optical filter. Direct-detection receiver may be the simplest receiver with low-cost implementation. However, direct-detection receiver for both phase- or frequency-modulated signal is still more complicated than a single photodiode to detect the presence or absence of light for on-off keying signal.

Another types of noncoherent receiver use phase-diversity techniques that combines two signals with a phase difference of 90°. Phase-diversity receiver is asynchronous receiver without the requirement of phase tracking. Equivalently speaking, phase-diversity receiver implements the envelope or power detection by combining optical and electrical techniques.

Other than polarization-diversity receivers, the mixing of two optical signals requires the alignment of their polarizations. In general, polarization alignment is provided by polarization controller. With optimal combining of the signal from both polarizations, polarization-diversity receivers have the same performance as receiver with polarization tracking.

Various types of coherent optical receiver will be discussed in this chapter.

### 1. Basic Coherent Receiver Structures

The basic structures of phase-shift keying (PSK) and differential phase-shift keying (DPSK) receivers have been shown in Figs. 1.3 and 1.4, respectively. This section studies each basic type of coherent optical receivers that mix the received signal with the LO laser. The signal-to-noise ratio (SNR) of each receiver type is derived, especially for systems limited by amplifier noises. The SNR is equal to the ratio of optical signal to the amplifier noise per polarization over an optical bandwidth equal to the data rate. For system without optical amplifiers, the SNR is equal to the number of photons per bit for heterodyne receiver.

## 1.1 Single-Branch Receiver

Figure 3.1 shows a typical structure of a single-branch coherent optical receiver. To enable optical signal mixing, the polarization of the received signal must be aligned to that of the LO laser. In Fig. 3.1, automatic polarization control (APC) is used to align the polarization of the received signal to that of the LO laser for optimal signal mixing. In general, the LO laser is phase or frequency locked to the received signal. Phase locking is used for homodyne receiver and frequency locking is required for heterodyne receiver for a fixed intermediate frequency (IF). Phase locking is facilitated by an optical PLL and frequency locking

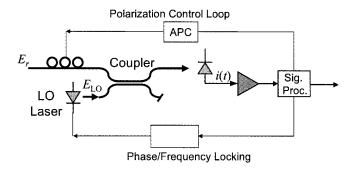


Figure 3.1. A single-branch coherent optical receiver.

is provided by an automatic frequency control (AFC) loop. The single-branch receiver is the simplest receiver structure. Most early heterodyne receivers used single-branch receiver for its simplicity (Goodwin, 1967, Nussmeier et al., 1974, Oliver, 1961, Peyton et al., 1972, Saito et al., 1981). Instead of 3-dB coupler, those early works used a power beam splitter or a power combiner, functioning as an 180° optical hybrid, to mix the received signal with the LO laser.

A 3-dB coupler is used in Fig. 3.1 to mix the received signal with the LO laser. The input and output relationship of a 3-dB coupler is<sup>1</sup>

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1\\ 1 & -1 \end{bmatrix}. \tag{3.1}$$

The received signal is assumed to be

$$E_r(t) = [A_s(t)e^{j\phi_s(t)} + n_x(t)]e^{j\omega_c t}\mathbf{x} + n_y(t)e^{j\omega_c t}\mathbf{y}, \tag{3.2}$$

where  $A_s(t)$  and  $\phi_s(t)$  are the modulated amplitude and phase of the transmitted signal, respectively,  $\omega_c$  is carrier frequency of the signal,  $\mathbf{x}$  is the polarization of the signal, and  $\mathbf{y}$  is the polarization orthogonal to  $\mathbf{x}$ ,  $n_x(t)$  and  $n_y(t)$  are the amplifier noise in the polarizations of  $\mathbf{x}$  and  $\mathbf{y}$ , respectively. The complex signal of  $A_s(t)e^{j\phi_s(t)}$  is the low-pass representation of the signal.  $A_s(t)e^{j\phi_s(t)} = \pm A$  for both PSK and DPSK signals

$$S = \frac{1}{\sqrt{2}} \left[ \begin{array}{cc} 1 & j \\ 1 & -j \end{array} \right].$$

without changing the phase relationship. Different input and output relationship for an  $180^{\circ}$  optical hybrid may be used in this book for convenience.

 $<sup>^1{\</sup>rm The~input}$  and output relationship can also be

and  $A_s(t)e^{j\phi_s(t)}=\{0,A\}$  for ASK signals, where A is the positive signal amplitude. Both  $n_x(t)$  and  $n_y(t)$  are the low-pass representation of the amplifier noise. Here, in this chapter, we assume that both the transmitted and LO lasers are a pure coherent source without phase noise. In next chapter, the impact of laser phase noise to phase-modulated signal is analyzed for further detail. The received signal may have a random phase of  $\theta_0$  of  $A_s(t)e^{j\phi_s(t)+j\theta_0}$  due to propagation delay. For system with phase tracking using PLL, we assume that  $\theta_0=0$  for simplicity when the phase of LO laser tracks out the phase of  $\theta_0$ . For system without phase tracking, the phase of  $\theta_0$  is included when necessary.

The optical SNR of the received signal is defined as

$$SNR_{o,s} = \frac{E\{|A_s(t)|^2\}}{E\{|n_x(t)|^2\} + E\{|n_y(t)|^2\}} = \frac{P_r}{2S_{n_s}\Delta f_{o,s}},$$
 (3.3)

where  $P_r$  is the received power,  $S_{n_s}$  is the spectral density of the received spontaneous emission in each polarization, and  $\Delta f_{o,s}$  is the optical bandwidth of the received optical filter. In the complex representation of  $n_x(t) = n_{x1}(t) + jn_{x2}(t)$ , the spectral densities of  $n_{x1}(t)$  and  $n_{x2}(t)$  are the same and equal to  $S_{n_s}/2$ .

The LO signal is

$$E_{\text{LO}}(t) = [A_L + n_L(t)]e^{j\omega_{\text{LO}}t}\mathbf{x}, \tag{3.4}$$

where  $A_L$  is the continuous-wave amplitude of LO laser,  $n_L(t)$  is the noise of LO laser in the same polarization as the signal, and  $\omega_{LO}$  is the angular frequency of LO laser. The polarization of the received signal of Eq. (3.2) and the LO laser of Eq. (3.4) is assumed to be the same using APC. The noise of  $n_L(t)$  may originate from the optical amplifier used to boost up the LO power or from the relative intensity noise (RIN) of the LO laser. In the LO electric field of Eq. (3.4), the noise at the polarization orthogonal to the signal is ignored here for simplicity. In practice, the noise from polarization orthogonal to the signal can be filtered by a polarizer, especially at a receiver having a well-controlled LO laser.

In the single-branch receiver of Fig. 3.1, the electric field at the input of the photodiode is  $[E_r(t) + E_{LO}(t)]/\sqrt{2}$ , the photocurrent is

$$i(t) = \frac{R}{2} |E_r(t) + E_{LO}(t)|^2 + i_{sh} + i_{th},$$
 (3.5)

where R is the responsivity of the photodiode,  $i_{\rm sh}$  is the photocurrent shot-noise, and  $i_{\rm th}$  is the thermal noise of the receiver. In later part of this chapter, other than specific, we ignored thermal noise for simplicity.

As an additive noise, the effect of thermal noise can be added to the signal afterward. In coherent optical communication systems with LO laser, thermal noise has less impact than both shot and amplifier noises.

The photodiode responsivity is equal to

$$R = \eta \frac{e}{\hbar \omega_c},\tag{3.6}$$

where  $e = 1.6 \times 10^{-19}$  C is the charge per electron,  $\hbar \omega_c$  is the energy per photon, where  $\hbar = h/(2\pi)$  with  $h = 6.63 \times 10^{-34}$  J s as the Planck constant,  $\eta$  is the quantum efficiency of photodiode that is the average number of electrons generated per a photon by the photodiode.

The photocurrent of Eq. (3.5) is equal to

$$i(t) = \frac{R}{2} \left\{ |A_L + n_L(t)|^2 + |A_s(t)e^{j\phi_s(t)} + n_x(t)|^2 + |n_y(t)|^2 \right\}$$

$$+ RA_L A_s(t) \cos[\omega_{\text{IF}} t + \phi_s(t)]$$

$$+ R\Re \left\{ \left[ A_L n_x(t) + A_s(t)e^{j\phi_s(t)} n_L(t) \right] e^{j\omega_{\text{IF}} t} \right\} + i_{\text{sh}}. (3.7)$$

In the photocurrent of Eq. (3.7), the intermediate frequency (IF) is  $\omega_{\rm IF} = \omega_c - \omega_{\rm LO}$ . Homodyne system has  $\omega_{\rm IF} = 0$  but heterodyne system has  $\omega_{\rm IF} \neq 0$ . The photocurrent of Eq. (3.7) includes the intensity of LO laser with noise of  $|A_L + n_L(t)|^2$ , the intensity of received signal with noise of  $|A_s(t)e^{j\phi_s(t)} + n_x(t)|^2$ , the intensity of spontaneous emission from orthogonal polarization of  $|n_y(t)|^2$ , the beating of received signal with LO laser of  $RA_LA_s(t)\cos[\omega_{\rm IF}t + \phi_s(t)]$ , LO and spontaneous beating of  $A_Ln_x(t)$ , signal and LO-spontaneous beating of  $A_s(t)n_L(t)$ , and together with shot noise  $i_{\rm sh}$ . In the photocurrent of Eq. (3.7), the small effect of the beating of spontaneous noise with shot noise is ignored. If necessary, thermal noise can add to the photocurrent of Eq. (3.7).

In the photocurrent of Eq. (3.7), the signal component is

$$s(t) = RA_L A_s(t) \cos[\omega_{IF} t + \phi_s(t)]. \tag{3.8}$$

For heterodyne system with  $\omega_{\rm IF} = \omega_{\rm c} - \omega_{\rm LO} \neq 0$ , the signal power is

$$P_s = \frac{R^2}{2} P_{\text{LO}} P_r, \tag{3.9}$$

where  $P_{\text{LO}} = A_L^2$  is the LO power,  $P_r = E\{|A_s(t)|^2\}$  is the received power. In PSK homodyne system with  $\omega_c = \omega_{\text{LO}}$ , we obtain

$$P_s = R^2 P_{\text{LO}} P_r. \tag{3.10}$$

Comparing the signal power of Eq. (3.9) for heterodyne system with the signal power of Eq. (3.10) for homodyne system, homodyne system has twice the signal power of heterodyne system. Conventionally, it was generally believed that homodyne system is 3-dB better than the corresponding heterodyne system (Betti et al., 1995, Hooijmans, 1994, Okoshi and Kikuchi, 1988, Ryu, 1995). However, for system dominated by optical amplifier noise, heterodyne system generally has the same performance as similar homodyne system.

The dominant noise source for the single-branch receiver depends on the system configuration. In the usual case that the LO power is significantly larger than the received power, i.e.,  $P_{\text{LO}} \gg P_r$ , the dominant noise is usually LO-spontaneous beating noise, given by  $R\Re\{A_L n_x(t)e^{j\omega_{\text{IF}}t}\}$  in the photocurrent of Eq. (3.7).

If the spontaneous emissions of LO and received signal are not negligible with  $n_L(t) \neq 0$  and  $n_x(t) \neq 0$ , the LO-spontaneous beating including two components of  $R\Re\{A_Ln_L(t)\}$  from  $\frac{1}{2}R|A_L+n_L(t)|^2$  and  $R\Re\{A_Ln_x(t)e^{j\omega_{\rm IF}t}\}$ . If the optical bandwidth of  $n_L(t)$  is  $\Delta f_{o,L}$  centered around  $\omega_L$ , the bandwidth of  $R\Re\{A_Ln_L(t)\}$  is about  $\Delta f_{o,L}/2$ . For heterodyne systems with  $f_{\rm IF} = \omega_{\rm IF}/(2\pi) \neq 0$  and  $f_{\rm IF} > \frac{1}{2}\Delta f_{o,L} + B_d$ , an electric filter can be designed such that the beating of  $A_L$  with  $n_L(t)$  does not affect the performance of the system, where  $B_d$  is the symbol rate of the data channel. In homodyne system with  $\omega_{\rm IF} = f_{\rm IF} = 0$ , the beating of  $A_L$  with  $n_L(t)$  always contributes to the noise of the system.

In the LO and spontaneous noise  $n_x(t)$  beating of  $R\Re\left\{A_Ln_x(t)e^{j\omega_{\rm IF}t}\right\}$ , if the optical bandwidth of  $n_x(t)$  is  $\Delta f_{o,s}$  centered around  $\omega_c$ , the upper and lower frequency of LO-spontaneous beating noise is  $f_{\rm IF} \pm \frac{1}{2}\Delta f_{o,s}$ . Figure 3.2 illustrates the effect of the optical filter bandwidth of  $\Delta f_{o,s}$  on the beating of LO laser and spontaneous emission of  $n_x(t)$ . The optical filter should have a center frequency align with the optical signal.

Without loss of generality and assume that  $f_{\rm IF} > 0$  as in Fig. 3.2, the lower frequency of  $f_{\rm IF} - \frac{1}{2}\Delta f_{o,s}$  may be a negative frequency for large  $\Delta f_{o,s}$ . The negative frequency noise affects the system if  $f_{\rm IF} - \frac{1}{2}\Delta f_{o,s}$  falls into the data bandwidth of  $-f_{\rm IF} + B_d$ .

The upper two traces of Fig. 3.2 show the case when the optical bandwidth of  $\Delta f_{o,s}$  is small and comparable to twice the data bandwidth of  $B_d$ . The beating noise is band-pass noise centered at  $f_{\rm IF}$ . The lower two traces of Fig. 3.2 show the case when the optical bandwidth of  $\Delta f_{o,s}$  is significantly larger than the data bandwidth of  $B_d$ . With  $f_{\rm IF} - \frac{1}{2}\Delta f_{o,s} < 0$ , the beating noise may extend to  $-f_{\rm IF} + B_d$ . In the worst case of having a wide-bandwidth receiver optical filter, the beating noise centered at  $f_{\rm IF}$  with a bandwidth of  $2B_d$  is doubled compared with case of narrow-bandwidth optical filter. In the diagram of Fig. 3.2, the data bandwidth is assumed to be  $2B_d$ . In practical system, depending on linecode or spectral filtering, the data bandwidth may vary.

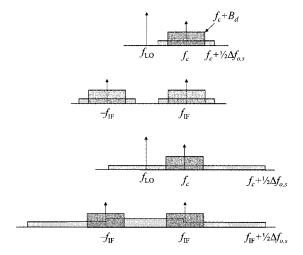


Figure 3.2. An illustration of the effects of receiver filter bandwidth of  $\Delta f_{o,s}$  on LO-spontaneous beat noise. The optical bandwidth of  $\Delta f_{o,s}$  is comparable (upper two traces) and larger than (lower two traces) than twice the data bandwidth.

The beating noise is not doubled if

$$-f_{\rm IF} + \frac{1}{2}\Delta f_{o,s} < f_{\rm IF} - B_d \tag{3.11}$$

or

$$f_{\rm IF} > \frac{1}{4} \left( \Delta f_{o,s} + 2B_d \right).$$
 (3.12)

In homodyne system, the LO and spontaneous noise  $n_x(t)$  beating is within the frequencies of  $\pm \Delta f_{o,s}/2$  and contributes to the positive and negative frequency noise twice. Because the power of homodyne receiver is twice that of heterodyne receiver by comparing Eq. (3.9) and (3.10), with the condition of Eq. (3.12), the output SNR of a single-branch homodyne and heterodyne receivers is the same. However, without the condition of Eq. (3.12), the noise of homodyne and heterodyne system is the same, homodyne system is 3 dB better than heterodyne system due to its higher power.

Under the condition of Eq. (3.12), heterodyne and homodyne systems have the same performance. With proper design, a coherent system with optical amplifiers is generally limited by the beating of LO-spontaneous emission beating between  $A_L$  and spontaneous emission of  $n_x(t)$ . When the optical filter has a bandwidth of  $\Delta f_{o,s} = 2B_d$ , the IF must be larger than  $f_{\rm IF} > B_d$ . In the limit, the narrowest optical bandwidth is actually

 $\Delta f_{o,s} = B_d$  for an optical match filter. However, for return-to-zero (RZ) signal with small duty cycle, the optical bandwidth of an optical match filter is larger than the data-rate of  $B_d$ .

In general, the condition of Eq. (3.12) is a good approximation for most practical cases. While the bandwidth of  $f_{\rm IF} \pm B_d$  is a good approximation, ideal sinc pulse has the bandwidth of  $f_{\rm IF} \pm B_d/2$  but RZ pulse has a bandwidth wider than  $2B_d$ .

Assuming the condition of Eq. (3.12), the LO-spontaneous beating noise is

$$RA_L n_{x1}(t)\cos(\omega_{\text{IF}}t) - RA_L n_{x2}(t)\sin(\omega_{\text{IF}}t), \tag{3.13}$$

where  $n_s(t) = n_{x1}(t) + jn_{x2}(t)$  with  $E\{n_{x1}^2(t)\} = E\{n_{x2}^2(t)\} = \frac{1}{2}S_{n_s}B_d$ . The LO laser source of  $A_L$  beating with  $n_x(t)$  induces an electrical noise with spectrum density of

$$N_{A_L - n_s} = \frac{1}{2} R^2 S_{n_s} P_{\text{LO}}.$$
 (3.14)

The LO laser source of  $A_L$  beating with  $n_L(t)$  induces electrical noise with spectrum density of

$$N_{A_L - n_L} = \frac{1}{2} R^2 S_{n_L} P_{\text{LO}}, \tag{3.15}$$

where  $S_{n_L}$  is the spectrum density of the spontaneous of  $n_L(t)$  at optical domain.

The optical SNR of LO laser source is

$$SNR_{o,LO} = \frac{P_{LO}}{2S_{n_L} \Delta f_{o,L}},$$
(3.16)

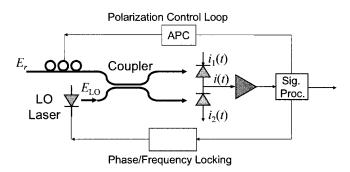
where the amplifier noise is in the same polarization as the signal and the orthogonal polarization, in contrast to the LO electric field of Eq. (3.4).

Because the receiver by itself should not induce noise into the system, the system must be designed for  $N_{A_L-n_s} > 10 \times N_{A_L-n_L}$ , or  $S_{n_s} > 10 \times S_{n_L}$ , where the factor of 10 is for the condition that the additional penalty due to  $n_L(t)$  is less than 0.5 dB. For optical SNR defined for the same bandwidth of  $\Delta f_{o,s} = \Delta f_{o,L}$ , the system requirement is

$$\frac{\text{SNR}_{o,\text{LO}}}{\text{SNR}_{o,s}} > 10 \times \frac{P_{\text{LO}}}{P_r},\tag{3.17}$$

with the condition of  $P_{LO} \gg P_r$ , the required optical SNR of the LO source must be far larger than that of the received signal.

If the optical SNR of LO source is not substantially larger than that of the signal source, the system is dominated by noise from the LO source.



 $Figure~3.3.~{\rm A~dual\mbox{-}photodiode~balanced~receiver~for~phase-modulated~optical~communications}.$ 

The noise of the LO source may be induced by an optical amplifier that boosts up the power of the LO laser or the RIN of the LO laser. In order to eliminate the effects of the LO laser noise, a balanced receiver can be used instead. Because balanced receiver has better performance than single-branch receiver, the analysis of this book usually assumes a balanced receiver.

In the receiver of Fig. 3.1, many methods can control the polarization though APC (Aarts and Khoe, 1989, Martinelli and Chipman, 2003, Noé et al., 1988a, 1991, Okoshi, 1985, Walker and Walker, 1990). The polarization control algorithm should be able to provide endless polarization control without reset. While early homodyne or heterodyne coherent optical communication systems used single-branch receiver, balanced receiver is more popular for its superior performance.

## 1.2 Balanced Receiver

Figure 3.3 shows a dual-photodiode balanced receiver that increases the signal power and eliminates the noise from the LO laser source. Similar to single-branch receiver of Fig. 3.1, balanced receiver of Fig. 3.3 requires both polarization alignment using APC and phase or frequency locking. The electric field at the input of the upper photodiode is  $[E_r(t) + E_{LO}(t)]/\sqrt{2}$ , the photocurrent is the same as Eq. (3.5) and is equal to

$$i_1(t) = \frac{R}{2} |E_r(t) + E_{LO}(t)|^2 + i_{sh1},$$
 (3.18)

where  $i_{\rm sh1}$  is the shot noise. The electric field at the input of the lower photodiode is  $[E_r(t) - E_{\rm LO}(t)]/\sqrt{2}$ , the photocurrent is

$$i_2(t) = \frac{R}{2} |E_r(t) - E_{LO}(t)|^2 + i_{sh2},$$
 (3.19)

where  $i_{\rm sh2}$  is the shot noise.

In both photocurrents of Eqs. (3.18) and (3.19), we assume that the two photodiodes are identical with the same responsivity and the 3-dB coupler or  $180^{\circ}$  optical hybrid is balanced 3-dB without excesses loss. The overall photocurrent is

$$i(t) = i_1(t) - i_2(t) = 2R\Re\{E_r(t) \cdot E_{LO}^*(t)\} + i_{sh}$$
 (3.20)

or

$$i(t) = 2RA_L A_s(t) \cos[\omega_{\text{IF}} t + \phi_s(t)]$$

$$+ 2R\Re\left\{ \left[ A_L n_x(t) + A_s(t) e^{j\phi_s(t)} n_L(t) \right] e^{j\omega_{\text{IF}} t} \right\} + i_{\text{sh}}, \quad (3.21)$$

where  $i_{\rm sh} = i_{\rm sh1} - i_{\rm sh2}$  is the overall shot noise.

In the photocurrent of Eq. (3.21), the noise from the LO laser of  $n_L(t)$  contributes to the system noise because of the beating term of  $2R\Re\{A_s(t)n_L(t)e^{j\omega_{\text{IF}}t+j\phi_s(t)}\}$  with a spectral density of

$$N_{A_s - n_L} = 2R^2 S_{n_L} P_r. (3.22)$$

Comparing with the LO-spontaneous beating noise of

$$N_{A_L - n_s} = 2R^2 S_{n_s} P_{LO}, (3.23)$$

as long as the optical SNR has the relationship of  $\mathrm{SNR}_{o,\mathrm{LO}} > 10 \times \mathrm{SNR}_{o,s}$  or  $N_{A_L-n_s} > 10 \times N_{A_s-n_L}$ , the signal and LO noise beating does not provide a penalty more than 0.5 dB. Even when a booster amplifier is used, the LO laser required low-gain optical amplifier but the signal usually passes through a chain of many optical amplifiers. The optical SNR of the LO signal is usually much larger than that of the received signal. The signal and spontaneous emission beating of  $N_{A_s-n_L}$  is usually much smaller than the LO-spontaneous emission beating of  $N_{A_L-n_s}$ .

The signal component of the photocurrent of Eq. (3.21) is

$$s(t) = 2RA_L A_s(t) \cos[\omega_{IF} t + \phi_s(t)] \tag{3.24}$$

with a signal amplitude twice of that in single-branch receiver of Eq. (3.8), giving four times larger received power. Since the noise is also increased by the same factor, balanced receiver does not increase the SNR for receiver limited by the beating of LO-spontaneous emission.

When the LO-spontaneous beating of  $2R\Re\{A_L n_x(t)e^{j\omega_{\text{IF}}t}\}$  is the dominant noise, similar to the illustration of Fig. 3.2 for single-branch receiver, a heterodyne balanced receiver also requires an optical filter

with the condition of Eq. (3.12) without doubling the LO-spontaneous beating noise. For systems limited by optical amplifier noise, the performance of heterodyne and homodyne receiver is the same with the condition of Eq. (3.12).

Here, the SNR of a heterodyne system is evaluated with amplifier noise and the condition of Eq. (3.12) for a balanced receiver, the signal power is

$$P_s = 2R^2 P_{\text{LO}} P_r. \tag{3.25}$$

The noise variance at a balanced receiver includes LO-spontaneous beating noise of

$$\sigma_{\text{LO-sp}}^2 = 2R^2 P_{\text{LO}} S_{n_s} B_d, \qquad (3.26)$$

and the signal-LO spontaneous beating noise variance of

$$\sigma_{\text{sig.-LOsp}}^2 = 2R^2 P_r S_{n_L} B_d. \tag{3.27}$$

The shot noise has a variance of

$$\sigma_{\rm sh}^2 = 2eR(P_{\rm LO} + P_r + 2S_{n_s}\Delta f_{o,s} + 2S_{n_L}\Delta f_{o,L})B_d$$
 (3.28)

for LO, signal, and spontaneous-emission induced shot noise.

For both heterodyne or homodyne systems, the SNR is

$$\rho_s = \frac{P_r}{S_{n_s} B_d + \frac{P_r}{P_{\rm LO}} S_{n_L} B_d + \frac{\hbar \omega_c}{\eta} \left( 1 + \frac{P_r}{P_{\rm LO}} \right) B_d},\tag{3.29}$$

when the shot noise of spontaneous emission and the beating of spontaneous emission with shot noise are ignored.

The LO spontaneous emission noise usually comes from a single amplifier with spectral density of

$$S_{n_T} = (G_L - 1)n_{\text{sp}L}\hbar\omega_{\text{LO}},\tag{3.30}$$

where  $G_L$  and  $n_{\text{sp}L}$  are the gain and spontaneous emission factor of the LO amplifier.

The spontaneous emission noise of  $S_{n_s}$  together with the received signal is induced by a chain of optical amplifiers in the fiber link. Assume  $N_A$  identical fiber spans with loss of  $1/G_s$ , equal to the gain of the  $N_A$  identical optical amplifiers, the first optical amplifier has an amplified spontaneous emission noise of

$$(G_s - 1)n_{\rm sps}\hbar\omega_c,\tag{3.31}$$

where  $n_{\rm sps}$  is the spontaneous emission factor of the optical amplifiers at each fiber span. The above spontaneous emission losses by the factor

of  $1/G_s$  at the fiber span, and is amplified by  $G_s$  by the second optical amplifier. The second optical amplifier also adds the same spontaneous emission as Eq. (3.31) to the optical signal. After  $N_A$  fiber spans, we obtain the overall amplifier noise spectral density of

$$S_{n_s} = N_A (G_s - 1) n_{\rm sps} \hbar \omega_c. \tag{3.32}$$

The noise figure of optical amplifier is approximately equal to  $2n_{\rm sps}$ . The spontaneous emission of Eq. (3.32) can be expressed using the noise figure of the optical amplifier as  $S_{n_s} \approx \frac{1}{2} G_s F_n \hbar \omega_c$ , where  $F_n$  is the noise figure of each optical amplifier.

#### Shot-noise limited systems

A heterodyne system without optical amplifiers is limited by the shot noise with  $n_s(t) = 0$  and  $n_L(t) = 0$ , the signal power is that of Eq. (3.25) and the noise variance is that of Eq. (3.28). If the LO laser has significantly larger power than the receive power of  $P_{\text{LO}} \gg P_r$ , the SNR of a heterodyne system becomes

$$\rho_s = \frac{RP_r}{eB_d}. (3.33)$$

For binary signal, if the average number of photons per bit is  $N_s$ ,  $P_r = N_s \hbar \omega_c B_d$ . For multilevel signal,  $N_s$  is the average number of photons per symbol. With the photodiode responsivity defined by Eq. (3.6), we obtain

$$\rho_s = \eta N_s. \tag{3.34}$$

In homodyne system, the power is twice that of Eq. (3.25) but the shot noise remains the same as that of Eq. (3.28), the SNR is

$$\rho_s = 2\eta N_s. \tag{3.35}$$

In the quantum limit with  $\eta = 1$ , the quantum-limited SNR is the photon number per bit of  $N_s$  and twice the photon number per bit of  $2N_s$  for heterodyne and homodyne systems, respectively. The SNR of Eqs. (3.34) and (3.35) is usually used in the analysis of traditional coherent optical communication systems limited by shot noise (Betti et al., 1995, Okoshi and Kikuchi, 1988).

#### Amplifier-noise limited systems

If the amplifier noise is the dominant noise source, with the condition of Eq. (3.12), the SNR after the balanced receiver for heterodyne system

with  $\omega_{\rm IF} \neq 0$  is

$$\rho_s = \frac{P_r}{S_{n_s} B_d}. (3.36)$$

The SNR of Eq. (3.36) has a very simple and clear physical meaning of the received signal of  $P_r$  over the optical noise within a bandwidth of  $B_d$  in a single polarization. As the spontaneous emission has a spectral density of  $S_{n_s}$ , the noise power per polarization is  $S_{n_s}B_d$ . In a RF system, with proper filtering, mixing by upconversion or downconversion does not change the SNR. From Eq. (3.36), the SNR in optical domain, before the downconversion by balanced receiver, is also the SNR in the receiver. The definition of the SNR of Eq. (3.36) ignores the amplifier noise from orthogonal polarization that does not beat with the signal. In the photocurrent of Eq. (3.21), the optical amplifier noise from orthogonal polarization does not affect the system.

For a received power of  $P_r = G_s N_s \hbar \omega_c B_d$ , we obtain

$$\rho_s = \frac{N_s}{N_A n_{\rm sps}} \frac{G_s}{G_s - 1} \approx \frac{N_s}{N_A n_{\rm sps}}.$$
 (3.37)

Comparing with Eq. (3.34), the SNR is degraded by the factor of  $N_A n_{\rm sps}$ . The SNR of Eq. (3.37) is valid for both homodyne and heterodyne systems. For heterodyne systems, the condition of Eq. (3.12) must be satisfied such that the LO-spontaneous emission does not double.

The equivalent spontaneous emission factor can be defined as

$$n_{\rm eq} = N_A n_{\rm sp} (G_s - 1) / G_s \approx \frac{1}{2} N_A F_n$$
 (3.38)

for  $N_A$  identical optical amplifiers where  $F_n$  is the noise figure of each optical amplifier. With the equivalent  $n_{\rm eq}$  of Eq. (3.38), the spectrum density of Eq. (3.32) becomes

$$S_{n_s} = (G_s - 1)n_{\text{eq}}\hbar\omega_c,\tag{3.39}$$

and the SNR is

$$\rho_s = \frac{N_s}{n_{eq}}. (3.40)$$

If the  $N_A$  optical amplifiers are not the same with different gain and noise figure, the equivalent spontaneous emission factor is

$$n_{\rm eq} = \sum_{k=1}^{N_A} \frac{P_{\rm in}N}{P_{\rm in}k} \frac{G_k - 1}{G_k} n_{{\rm sp}s_k}, \tag{3.41}$$

where  $G_k$ ,  $k = 1, ..., N_A$ , are the gain of each optical amplifier,  $P_{\text{in}k}$ ,  $k = 1, ..., N_A$ , are the input power of each optical amplifier, and  $n_{\text{sp}s_k}$  are the

spontaneous emission factor of each optical amplifier. If all amplifiers are identical with  $G_k = G_s$  and  $n_{\text{sp}s_k} = n_{\text{sp}s}$ , we obtain  $n_{\text{eq}} = N_A n_{\text{sp}s} (G_s - 1)/G_s$  of Eq. (3.38) again.

The SNR of Eq. (3.40) is the same as the optical SNR defined over a bandwidth of  $B_d$  for a single polarization of

$$\frac{P_r}{S_{n_s}B_d} = \frac{N_s}{n_{\text{eq}}}. (3.42)$$

In the quantum limit of a single optical amplifier of  $N_A = 1$  and  $F_n = 2$  (or 3 dB),  $n_{\text{eq}} = 1$ , the quantum limited SNR is equal to the average number of photons per bit of  $N_s$ , the same as that of shot-noise limited heterodyne system.

In practice, optical SNR of SNR<sub>o,s</sub> is measured over an optical bandwidth of  $\Delta f_{o,s}$  using an optical spectrum analyzer. The SNR of Eq. (3.40) is equal to

$$\rho_s = \text{SNR}o, s \frac{2\Delta f_{o,s}}{B_d}, \tag{3.43}$$

where the factor of 2 is for two polarizations in most optical SNR measurement. The optical bandwidth in typical measurements is  $\Delta f_{o,s} = 12.5 \text{ GHz}$ , corresponding to 0.1 nm or 1 Å in the wavelength around 1.55  $\mu$ m. The difference between  $\rho_s$  and SNR<sub>o,s</sub> is about 4.0 and -2.0 dB for 10 and 40-Gb/s signals, respectively.

If no optical filter is used or the optical filter has a wide bandwidth that does not conform to the condition of Eq. (3.12), the amplifier-noise limited SNR for heterodyne receiver is

$$\rho_s = \frac{N_s}{2n_{\rm eq}}. (3.44)$$

In later section, we always assume a SNR of Eq. (3.40) for heterodyne receiver. With a balanced receiver, the performance of a system depends solely on the SNR. Both heterodyne and homodyne systems have the same performance if the SNR of the system is the same.

Balanced receiver for coherent optical communications was first analyzed in detail by Abbas et al. (1985), Yuen and Chan (1983), and Alexander (1987) to suppress the LO noise and to obtain signal power gain. Without LO noise and ignored thermal noise, the performance of balanced receiver should be the same as that of a single-branch receiver.

## 1.3 Quadrature Receiver

Figure 3.4 shows a quadrature receiver to recover both the in- and quadrature-phase components of the optical signal. The quadrature re-

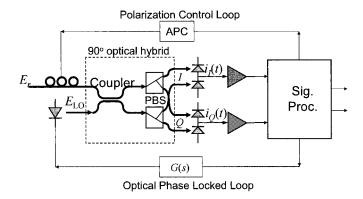


Figure 3.4. A quadrature homodyne receiver.

ceiver based on a 90° optical hybrid and two balanced receivers. Although a single-branch receiver of Fig. 3.1 can be used in Fig. 3.4, a balanced receiver has better performance, especially in the presence of LO laser noise.

The  $90^\circ$  optical hybrid composites of a 3-dB coupler and two polarization beam splitters (PBS). Using the two PBS as the reference polarization, the received signal must be controlled to be linearly polarized with a direction  $45^\circ$  from the PBS reference polarization. The received signal excluding noise is

$$E_r = \frac{1}{\sqrt{2}} (\mathbf{x} + \mathbf{y}) A_s(t) e^{j\phi_s(t) + j\omega_c t}.$$
 (3.45)

The LO laser must be circular polarized with an electric field of

$$E_{\text{LO}} = \frac{1}{\sqrt{2}} (\mathbf{x} + e^{j\pi/2} \mathbf{y}) A_L e^{j\omega_{\text{LO}}t}.$$
 (3.46)

At the output of the 3-dB coupler, the two electric fields before the PBS are

$$E_{1} = \frac{1}{\sqrt{2}} (E_{r} + E_{LO})$$

$$= \frac{\mathbf{x}}{2} \underbrace{\left[ A_{s}(t)e^{j\phi_{s}(t) + j\omega_{c}t} + A_{L}e^{j\omega_{LO}t} \right]}_{0^{\circ}} + \frac{\mathbf{y}}{2} \underbrace{\left[ A_{s}(t)e^{j\phi_{s}(t) + j\omega_{c}t} + A_{L}e^{j\omega_{LO}t + j\pi/2} \right]}_{90^{\circ}}, \quad (3.47)$$

and

$$E_{2} = \frac{1}{\sqrt{2}} (E_{r} - E_{LO})$$

$$= \frac{\mathbf{x}}{2} \left[ A_{s}(t)e^{j\phi_{s}(t) + j\omega_{c}t} - A_{L}e^{j\omega_{LO}t} \right]$$

$$+ \frac{\mathbf{y}}{2} \left[ A_{s}(t)e^{j\phi_{s}(t) + j\omega_{c}t} - A_{L}e^{j\omega_{LO}t + j\pi/2} \right]. \quad (3.48)$$

The two PBS separate the above two electric fields to the polarization directions of  $\mathbf{x}$  and  $\mathbf{y}$ . The upper balanced receiver combines the photocurrents corresponding to both  $0^{\circ}$  and  $180^{\circ}$ , the received photocurrent is

$$i_{I}(t) = \frac{R}{4} \left| A_{s}(t)e^{j\phi_{s}(t)+j\omega_{c}t} + A_{L}e^{j\omega_{LO}t} \right|^{2}$$

$$-\frac{R}{4} \left| A_{s}(t)e^{j\phi_{s}(t)+j\omega_{c}t} - A_{L}e^{j\omega_{LO}t} \right|^{2}$$

$$= RA_{s}(t)A_{L}\cos[\omega_{IF}t + \phi_{s}(t)]. \tag{3.49}$$

The lower balanced receiver combines the photocurrents corresponding to both 90° and 270°, the received photocurrent is

$$i_{Q}(t) = \frac{R}{4} \left| A_{s}(t)e^{j\phi_{s}(t)+j\omega_{c}t} + A_{L}e^{j\omega_{LO}t+j\pi/2} \right|^{2}$$

$$-\frac{R}{4} \left| A_{s}(t)e^{j\phi_{s}(t)+j\omega_{c}t} - A_{L}e^{j\omega_{LO}t+j\pi/2} \right|^{2}$$

$$= RA_{s}(t)A_{L}\sin[\omega_{IF}t + \phi_{s}(t)].$$
(3.50)

In both photocurrents of Eqs. (3.49) and (3.50), an nonzero IF frequency of  $\omega_{\rm IF}$  is assumed for a general quadrature receiver. Homodyne quadrature receiver has  $\omega_{\rm IF}=0$ .

Mathematically, a  $2 \times 2~90^{\circ}$  optical hybrid has an input and output relationship of

$$S = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & j \end{bmatrix}. \tag{3.51}$$

and a  $2 \times 4~90^{\circ}$  optical hybrid has an input and output relationship of

$$S = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & j \\ 1 & -1 \\ 1 & -j \end{bmatrix}. \tag{3.52}$$

For a homodyne receiver with a PLL of Fig. 3.4,  $\omega_{\text{IF}} = 0$  and  $i_I(t)$  gives the in-phase component of  $A_s(t)\cos\phi_s(t)$ , and  $i_Q(t)$  provides the quadrature-phase component of  $A_s(t)\sin\phi_s(t)$ . The quadrature homodyne receiver can be used as the receiver of both M-ary PSK and quadrature-amplitude modulation (QAM) signals.

In the homodyne quadrature receiver of Fig. 3.4, the shot-noise limited SNR is the same as that in Eq. (3.35) and the amplifier-noise limited SNR is that of Eq. (3.40). The noises in the in- and quadrature-phase components are independent of each other. The SNR for the photocurrents of Eqs. (3.49) and (3.50) is identical.

For a heterodyne quadrature receiver with  $\omega_{\text{IF}} \neq 0$ ,  $i_I(t)$  and  $i_Q(t)$  are quadrature components with a phase difference of 90°. With the condition of Eq. (3.12), the amplifier-noise limited SNR is that of Eq. (3.40). The shot-noise limited SNR is that of Eq. (3.34).

In heterodyne receiver, it may be more convenient to use electrical PLL to obtain both in- and quadrature-phase components. In another application as shown later, a heterodyne quadrature receiver similar to Fig. 3.4 can be used to provide image-rejection.

In- and quadrature-phase detection was first used to simultaneously measure phase and amplitude of an optical electric field (Walker and Carroll, 1984). The first application to coherent optical communications was by Hodgkinson et al. (1985). The 90° optical hybrid of Fig. 3.4 is designed according to Kazovsky et al. (1987). Other implementations of 90° optical hybrid were proposed by Delavaux and Riggs (1990), Delavaux et al. (1990), Hoffman et al. (1989), and Langenhorst et al. (1991). Recently, Cho et al. (2004c) shown an integrated LiNbO<sub>3</sub> optical 90° hybrid. As shown later, both image-rejection and phase-diversity receivers are similar to the quadrature receiver of Fig. 3.4. Recently, without phase-locking, quadrature receivers similar to Fig. 3.4 were used for measurement purpose (Dorrer et al., 2003, 2005).

# 1.4 Image-Rejection Heterodyne Receiver

In a densely space coherent wavelength-division-multiplexed (WDM) systems, the most important issue is to improve the spectral efficiency by allotting more channels within the amplifier passband of the system. In the heterodyne receiver with  $\omega_{\rm IF} \neq 0$  in both Figs. 3.1 and 3.3, the signals at both  $\omega_{c1}$  and  $\omega_{c2}$  fall to the same IF band if  $\omega_{\rm IF} = \omega_{c1} - \omega_{\rm LO} = \omega_{\rm LO} - \omega_{c2}$ . In a regular balanced heterodyne receiver of Fig. 3.3, a large guard-band of about  $2\omega_{\rm IF}$  is required such that the real- and imageband signals do not interfere with each other. The requirement of large guard-band limits the spectral efficiency of the system.